Supplementary Materials for Yuan, Zhang, and Deng

October 4, 2017

1 Using WebSEM for Data Analysis

The fit indices for mean structures with growth curve models developed in this article are currently available in WebSEM, which is free online software that allows researchers to conduct SEM analysis through path diagrams (Zhang & Yuan, 2012-2017).[1] In this section, we illustrate how to obtain the fit indices in WebSEM through the linear model with equal error variances in Example 4 used in the paper. A video with instructions on how to conduct the data analysis is also available on YouTube (`https://youtu.be/1ydG3IxBIsM`).

To use WebSEM, one needs to provide a data file and draw a path diagram. The data file should be a free format text file with the extension name txt. The data file can be generated using any text editor or the default editor within WebSEM. Both raw data and summary data can be used. For missing data and non-normal data analysis, raw data are required. If raw data are used, the first line of the data file should be variable names and the rest are data entries. If summary data are used, the first row includes the variable names. The second row provides the value of the sample size that is repeated for each column of data. The third row includes information on the sample means. The rest is the full sample covariance matrix. For example, the summary data in the data file for Example 4 from Preacher et al. (2008) are given below. Clearly, there are six variables named close1, close2, close3, close4, close5 and gender, respectively, and the sample size is 851. Note that the value 851 repeats itself 6 times.

close1	close2	close3	close4	close5	gender
851	851	851	851	851	851
37.9542	37.2785	37.0463	36.5696	36.1363	0.4900
6.3944	3.2716	4.1435	3.7058	4.1286	-0.0940
3.2716	7.5282	6.0804	5.1597	5.7608	-0.0390
4.1435	6.0804	10.7290	6.5672	7.2365	-0.1521
3.7058	5.1597	6.5672	10.2920	7.6463	-0.1104
4.1286	5.7608	7.2365	7.6463	12.9085	-0.1469
-0.0940	-0.0390	-0.1521	-0.1104	-0.1469	0.2502

WebSEM provides an intuitive and interactive interface to draw path diagram for a model for data analysis. Information on how to use the interface can be found in Zhang and Yuan (2012-2017). The path diagram for the linear growth curve model drawn by WebSEM is given in Figure 2. The video on YouTube showed how to draw a path diagram interactively. Because the use of summary data instead of raw data, we put "`data=cov`" in the control field of WebSEM. Furthermore, to request the fit indices for the mean structure, type "`mean.fit`" in the control field; otherwise, only fit indices for the combined mean and covariance structure will be produced.

The output of WebSEM by running the analysis is shown below. Test Statistics and Fit Indices for both mean and covariance, covariance only, and mean only are given at the beginning of the output. For example, for evaluating both mean and covariance together, the chi-square statistic is 81.653 with 17 degrees of freedom. The corresponding p-value is essentially 0. The CFI is 0.971, NFI is 0.964, NNFI is 0.961 and RMSEA is 0.067. For mean only, the chi-square statistics is 5.073 (p-value = 0.167) and RMSEA is 0.028.

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[1] Zhang, Z. & Yuan, K.-H. (2012-2017). WebSEM: Structural equation modeling online [Manual available at http://psychstat.org/websem]. Retrievable from https://websem.psychstat.org.
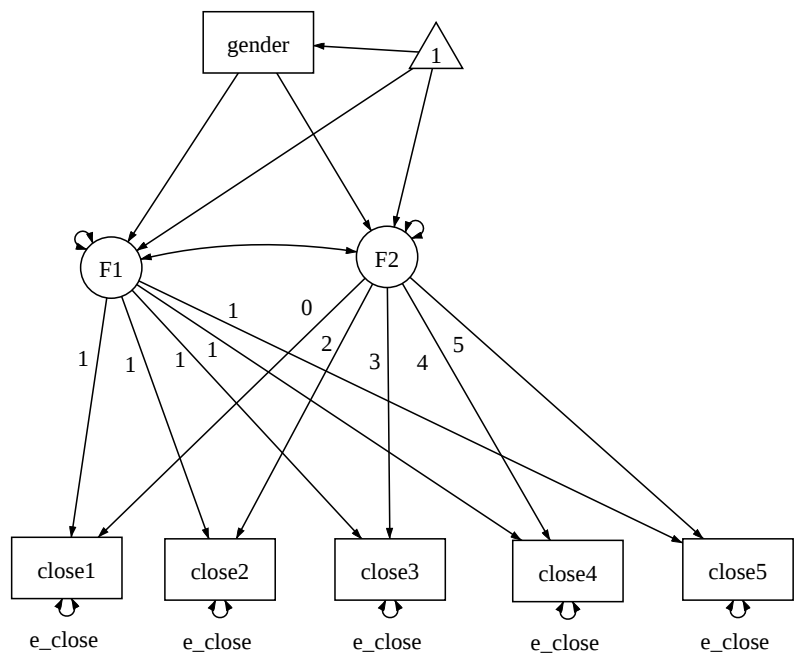
Figure 1: Path diagram for the linear growth curve model with equal error variances. The path diagram was drawn by WebSEM.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    81.653
        Degrees of freedom      17
        p-value          0
        Chi-square statistic (baseline model)  2272.4
        Degrees of freedom (baseline model)    23
        CFI             0.971
        NFI             0.964
        NNFI            0.961
        RMSEA           0.067
Fit for covariance only
        Chi-square statistic    76.586
        Degrees of freedom      14
        p-value          0
        Chi-square statistic (baseline model)  1971.2
        Degrees of freedom (baseline model)    19
        CFI             0.968
        NFI             0.961
        NNFI            0.956
        RMSEA           0.072
Fit for mean only
        Chi-square statistic    5.073
        Degrees of freedom      3
        p-value          0.167
        Chi-square statistic (baseline model)  287.88
        Degrees of freedom (baseline model)    4
        CFI             0.993
        NFI             0.982
        NNFI            0.99
        RMSEA           0.028
Parameter Estimates
                    Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1              1.000
    close3              1.000
    close4              1.000
    close5              1.000
    close6              1.000
  F2 =~
    close1              0.000
    close3              2.000
    close4              3.000
    close5              4.000
    close6              5.000

Regressions:
  F1 ~
    gender             -0.289    0.163   -1.770    0.077
  F2 ~
    gender             -0.052    0.043   -1.216    0.224
```

```
Covariances:
  F1 ~~
    F2                     0.249     0.056     4.420     0.000

Intercepts:
    F2                    -0.333     0.030   -11.169     0.000
    F1                    38.141     0.114   334.196     0.000
    close1                 0.000
    close3                 0.000
    close4                 0.000
    close5                 0.000
    close6                 0.000

Variances:
    close1   (e_y)         3.696     0.103    35.728     0.000
    close3   (e_y)         3.696     0.103    35.728     0.000
    close4   (e_y)         3.696     0.103    35.728     0.000
    close5   (e_y)         3.696     0.103    35.728     0.000
    close6   (e_y)         3.696     0.103    35.728     0.000
    F1                     2.956     0.284    10.399     0.000
    F2                     0.136     0.020     6.804     0.000
```

We now include the input and output of the examples used in our paper.

1.1 Example 1

The summary data used in this example are given below. The model in Figure 2 is used.

```
y1      y2      y3      y4      y5      y6
200     200     200     200     200     200
1.0     1.5     2.0     2.5     3.0     3.5
2.0     2.0     2.0     2.5     3.0     3.5
2.0     4.0     4.5     6.0     7.5     9.0
2.0     4.5     8.0    10.0    12.0    14.5
2.5     6.0    10.0    14.0    16.5    20.0
3.0     7.5    12.0    16.5    22.0    26.0
3.5     9.0    14.5    20.0    26.0    32.0
```
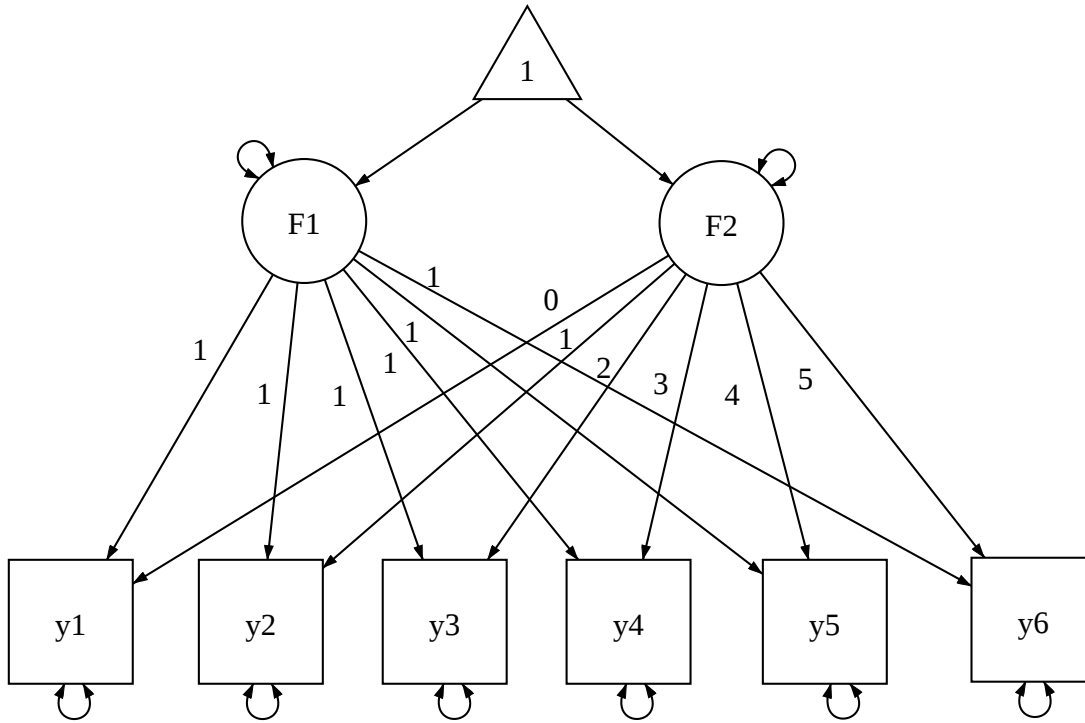
Figure 2: Path diagrams for models used in Examples 1-4

The output of WebSEM by running the analysis is shown below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic   82.686
        Degrees of freedom     16
        p-value          0
        Chi-square statistic (baseline model)  2010.2
        Degrees of freedom (baseline model)    20
        CFI            0.966
        NFI            0.959
        NNFI           0.958
        RMSEA          0.144
Fit for covariance only
        Chi-square statistic   82.686
        Degrees of freedom     12
        p-value          0
        Chi-square statistic (baseline model)  1923.8
        Degrees of freedom (baseline model)    15
        CFI            0.963
        NFI            0.957
        NNFI           0.954
        RMSEA          0.172
Fit for mean only
        Chi-square statistic   0
        Degrees of freedom     4
        p-value          1
        Chi-square statistic (baseline model)  46.273
```

```
        Degrees of freedom (baseline model)     5
        CFI                1
        NFI                1
        NNFI             1.121
        RMSEA              0

Parameter Estimates
                     Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    y1                 1.000
    y2                 1.000
    y3                 1.000
    y4                 1.000
    y5                 1.000
  F2 =~
    y1                 0.000
    y2                 1.000
    y3                 2.000
    y4                 3.000
    y5                 4.000
  F1 =~
    y6                 1.000
  F2 =~
    y6                 5.000

Covariances:
  F1 ~~
    F2     (r_F1)      0.378     0.100     3.779     0.000

Intercepts:
    F2                 0.500     0.073     6.819     0.000
    F1                 1.000     0.094    10.667     0.000
    y1                 0.000
    y2                 0.000
    y3                 0.000
    y4                 0.000
    y5                 0.000
    y6                 0.000

Variances:
    y1                 0.532     0.103     5.186     0.000
    y2                 0.642     0.085     7.590     0.000
    y3                 1.040     0.119     8.740     0.000
    y4                 1.087     0.125     8.715     0.000
    y5                 0.595     0.090     6.627     0.000
    F1                 1.444     0.181     7.970     0.000
    F2                 1.044     0.108     9.710     0.000
    y6                 0.483     0.127     3.807     0.000
```

1.2 Example 2

The summary data used in this example are given below. The same model in Figure 2 is used.

```
y1      y2      y3      y4      y5      y6
200.0   200.0   200.0   200.0   200.0   200.0
1.0     1.5     2.0     2.5     3.0     3.5
2.0     2.2     2.0     2.5     3.0     3.5
2.2     4.0     4.5     6.0     7.5     9.0
2.0     4.5     8.0     10.2    12.0    14.5
2.5     6.0     10.2    14.0    16.5    20.0
3.0     7.5     12.0    16.5    22.0    26.2
3.5     9.0     14.5    20.0    26.2    32.0
```

The output of the analysis is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    188.53
        Degrees of freedom      16
        p-value         0
        Chi-square statistic (baseline model)   2255.5
        Degrees of freedom (baseline model)     20
        CFI             0.923
        NFI             0.916
        NNFI            0.904
        RMSEA           0.232
 Fit for covariance only
        Chi-square statistic    188.53
        Degrees of freedom      12
        p-value         0
        Chi-square statistic (baseline model)   2168.6
        Degrees of freedom (baseline model)     15
        CFI             0.918
        NFI             0.913
        NNFI            0.898
        RMSEA           0.271
 Fit for mean only
        Chi-square statistic    0
        Degrees of freedom      4
        p-value         1
        Chi-square statistic (baseline model)   46.193
        Degrees of freedom (baseline model)     5
        CFI             1
        NFI             1
        NNFI            1.122
        RMSEA           0
Parameter Estimates
                    Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    y1                  1.000
    y2                  1.000
    y3                  1.000
    y4                  1.000
    y5                  1.000
  F2 =~
    y1                  0.000
    y2                  1.000
```

```
      y3                  2.000
      y4                  3.000
      y5                  4.000
  F1 =~
      y6                  1.000
  F2 =~
      y6                  5.000

Covariances:
  F1 ~~
      F2      (r_F1)     0.326      0.103      3.160      0.002

Intercepts:
      F2                 0.500      0.074      6.797      0.000
      F1                 1.000      0.097     10.321      0.000
      y1                 0.000
      y2                 0.000
      y3                 0.000
      y4                 0.000
      y5                 0.000
      y6                 0.000

Variances:
      y1                 0.288      0.073      3.957      0.000
      y2                 0.438      0.058      7.525      0.000
      y3                 1.188      0.130      9.115      0.000
      y4                 1.226      0.134      9.180      0.000
      y5                 0.405      0.064      6.311      0.000
      F1                 1.687      0.192      8.793      0.000
      F2                 1.065      0.108      9.838      0.000
      y6                 0.255      0.093      2.751      0.006
```

1.3 Example 3

The summary data for Example 3 are given below.

```
y1      y2      y3      y4      y5      y6
200.0   200.0   200.0   200.0   200.0   200.0
1.2     1.5     2.0     2.5     3.0     3.2
2.0     1.5     2.0     2.5     3.0     3.5
1.5     4.0     4.5     6.0     7.5     9.0
2.0     4.5     8.0     9.5    12.0    14.5
2.5     6.0     9.5    14.0    16.5    20.0
3.0     7.5    12.0    16.5    22.0    25.5
3.5     9.0    14.5    20.0    25.5    32.0
```

The linear growth curve model in Figure 2 is first fitted to the data and the output is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    33.093
        Degrees of freedom        16
        p-value            7e-03
        Chi-square statistic (baseline model)   1791.9
        Degrees of freedom (baseline model)       20
```

8

```
        CFI              0.99
        NFI              0.982
        NNFI             0.988
        RMSEA            0.073
Fit for covariance only
        Chi-square statistic   13.121
        Degrees of freedom     12
        p-value          0.36
        Chi-square statistic (baseline model)  1650.6
        Degrees of freedom (baseline model)    15
        CFI              0.999
        NFI              0.992
        NNFI             0.999
        RMSEA            0.022
Fit for mean only
        Chi-square statistic   20.073
        Degrees of freedom     4
        p-value          0
        Chi-square statistic (baseline model)  99.134
        Degrees of freedom (baseline model)    5
        CFI              0.829
        NFI              0.798
        NNFI             0.787
        RMSEA            0.142
Parameter Estimates
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    y1               1.000
    y2               1.000
    y3               1.000
    y4               1.000
    y5               1.000
  F2 =~
    y1               0.000
    y2               1.000
    y3               2.000
    y4               3.000
    y5               4.000
  F1 =~
    y6               1.000
  F2 =~
    y6               5.000


Covariances:
  F1 ~~
    F2               0.499    0.094    5.327    0.000


Intercepts:
    F2               0.429    0.073    5.902    0.000
    F1               1.162    0.087   13.304    0.000
    y1               0.000
    y2               0.000
    y3               0.000
```

```
      y4                    0.000
      y5                    0.000
      y6                    0.000

Variances:
      y1                    0.999    0.146    6.819    0.000
      y2                    1.007    0.122    8.223    0.000
      y3                    0.994    0.116    8.599    0.000
      y4                    0.995    0.122    8.127    0.000
      y5                    1.018    0.151    6.761    0.000
      F1                    0.991    0.158    6.266    0.000
      F2                    0.994    0.105    9.440    0.000
      y6                    1.024    0.204    5.015    0.000
```

The nonlinear growth curve model in Figure 3 below is then fitted to the data and the output is given below.
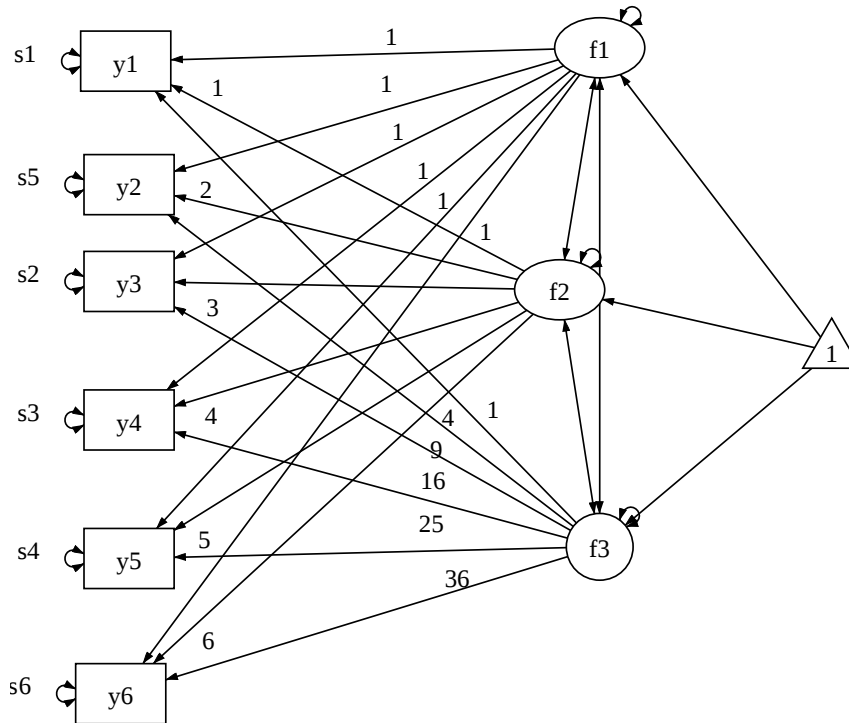

Figure 3: Qudratic growth curve model with unequal variances and a covariate

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic   4.849
        Degrees of freedom     12
        p-value          0.963
```

```
          Chi-square statistic (baseline model)   1791.9
          Degrees of freedom (baseline model)     20
          CFI              1
          NFI              0.997
          NNFI             1.007
          RMSEA            0
Fit for covariance only
          Chi-square statistic    0.814
          Degrees of freedom      9
          p-value          1
          Chi-square statistic (baseline model)   1650.6
          Degrees of freedom (baseline model)     15
          CFI              1
          NFI              1
          NNFI             1.008
          RMSEA            0
Fit for mean only
          Chi-square statistic    4.056
          Degrees of freedom      3
          p-value          0.256
          Chi-square statistic (baseline model)   91.106
          Degrees of freedom (baseline model)     5
          CFI              0.988
          NFI              0.955
          NNFI             0.98
          RMSEA            0.042
Parameter Estimates
                    Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    y1                 1.000
    y2                 1.000
    y3                 1.000
    y4                 1.000
    y5                 1.000
  F2 =~
    y1                 0.000
    y2                 1.000
    y3                 2.079     0.178    11.705     0.000
    y4                 3.159     0.276    11.435     0.000
    y5                 4.239     0.380    11.168     0.000
  F1 =~
    y6                 1.000
  F2 =~
    y6                 5.193     0.472    10.993     0.000

Covariances:
  F1 ~~
    F2                 0.490     0.092     5.313     0.000

Intercepts:
    F2                 0.410     0.080     5.143     0.000
    F1                 1.165     0.090    12.987     0.000
    y1                 0.000
```

```
    y2                    0.000
    y3                    0.000
    y4                    0.000
    y5                    0.000
    y6                    0.000

Variances:
    y1                    1.003    0.147    6.827    0.000
    y2                    1.002    0.122    8.180    0.000
    y3                    0.994    0.116    8.600    0.000
    y4                    0.990    0.122    8.085    0.000
    y5                    0.989    0.152    6.490    0.000
    F1                    1.005    0.165    6.081    0.000
    F2                    0.899    0.200    4.493    0.000
    y6                    1.064    0.208    5.115    0.000
```

1.4 Example 4

The summary data for Example 4 are given below.

```
close1    close2    close3    close4    close5    gender
851       851       851       851       851       851
37.9542   37.2785   37.0463   36.5696   36.1363   0.4900
6.3944    3.2716    4.1435    3.7058    4.1286   -0.0940
3.2716    7.5282    6.0804    5.1597    5.7608   -0.0390
4.1435    6.0804    10.7290   6.5672    7.2365   -0.1521
3.7058    5.1597    6.5672    10.2920   7.6463   -0.1104
4.1286    5.7608    7.2365    7.6463    12.9085  -0.1469
-0.0940  -0.0390   -0.1521   -0.1104   -0.1469   0.2502
```

4.1. The linear growth curve model with homogeneous variance in Figure 4 is first fitted to the data and the output is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    81.647
        Degrees of freedom      17
        p-value         0
        Chi-square statistic (baseline model)   9448.2
        Degrees of freedom (baseline model)     20
        CFI             0.993
        NFI             0.991
        NNFI            0.992
        RMSEA           0.067
Fit for covariance only
        Chi-square statistic    76.586
        Degrees of freedom      13
        p-value         0
        Chi-square statistic (baseline model)   1971.2
        Degrees of freedom (baseline model)     15
        CFI             0.967
        NFI             0.961
        NNFI            0.962
        RMSEA           0.076
Fit for mean only
```
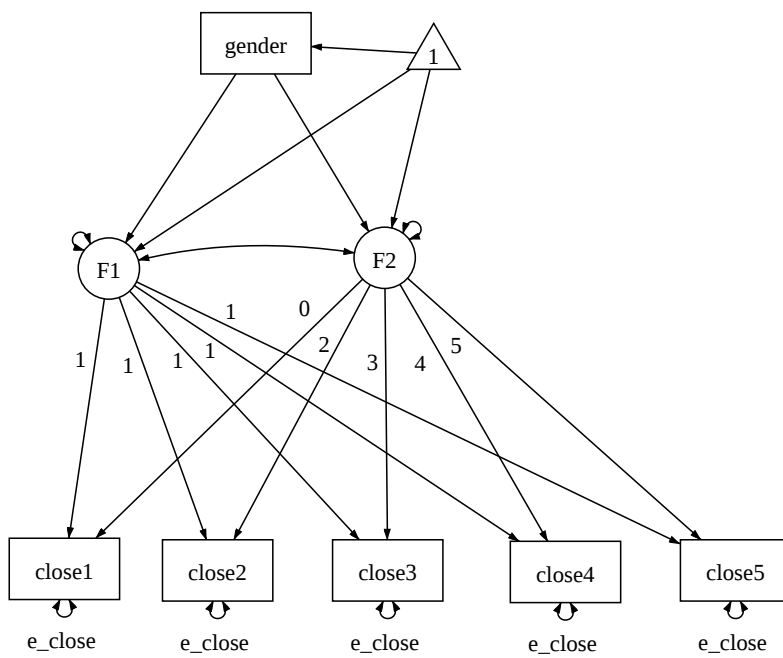
Figure 4: Linear growth curve model with equal variances and a covariate

```
         Chi-square statistic    821.71
         Degrees of freedom     4
         p-value          0
         Chi-square statistic (baseline model)   209969
         Degrees of freedom (baseline model)    5
         CFI              0.996
         NFI              0.996
         NNFI             0.995
         RMSEA            0.49
Parameter Estimates
                  Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1            1.000
    close2            1.000
    close3            1.000
    close4            1.000
    close5            1.000
  F2 =~
    close1            0.000
    close2            2.000
    close3            3.000
    close4            4.000
    close5            5.000

Regressions:
  F1 ~
    gendr (b__F1)    -0.289     0.163    -1.769     0.077
  F2 ~
    gendr (b__F2)    -0.052     0.043    -1.215     0.224

Covariances:
  F1 ~~
    F2      (r_F1)    0.250     0.057     4.417     0.000

Intercepts:
    F1      (a_F1)   38.141     0.114   333.899     0.000
    F2      (a_F2)   -0.333     0.030   -11.159     0.000
    gender           0.490     0.017    28.560     0.000
    close1           0.000
    close2           0.000
    close3           0.000
    close4           0.000
    close5           0.000

Variances:
    F1      (e_F1)    2.960     0.285    10.393     0.000
    F2      (e_F2)    0.136     0.020     6.800     0.000
    close1 (e_cl)    3.700     0.104    35.707     0.000
    close2 (e_cl)    3.700     0.104    35.707     0.000
    close3 (e_cl)    3.700     0.104    35.707     0.000
    close4 (e_cl)    3.700     0.104    35.707     0.000
    close5 (e_cl)    3.700     0.104    35.707     0.000
    gender           0.250     0.012    20.616     0.000
```
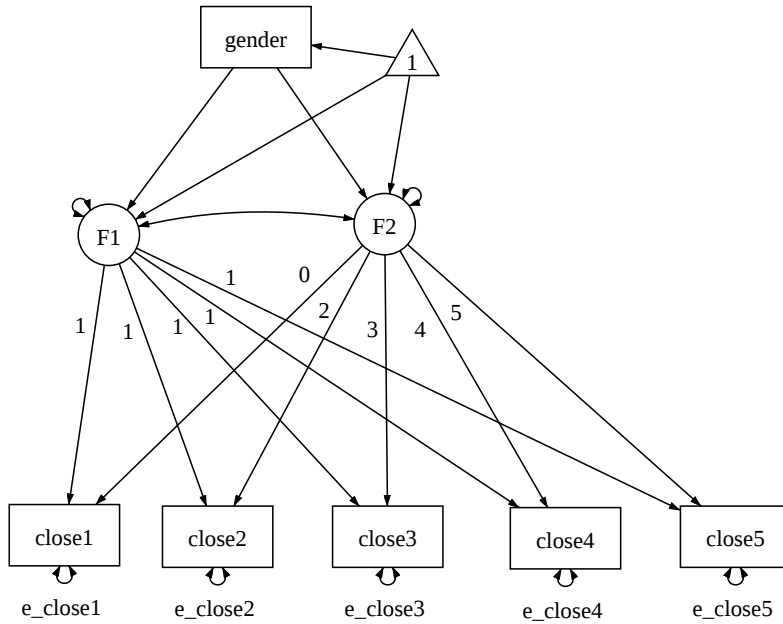
Figure 5: Linear growth curve model with unequal variances and a covariate

4.2. Linear model with unequal variances.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    58.457
        Degrees of freedom      13
        p-value          0
        Chi-square statistic (baseline model)  9448.2
        Degrees of freedom (baseline model)    20
        CFI            0.995
        NFI            0.994
        NNFI           0.993
        RMSEA          0.064
Fit for covariance only
        Chi-square statistic    53.862
        Degrees of freedom      9
        p-value          0
        Chi-square statistic (baseline model)  1971.2
        Degrees of freedom (baseline model)    15
        CFI            0.977
        NFI            0.973
        NNFI           0.962
        RMSEA          0.077
Fit for mean only
        Chi-square statistic    821.25
```

```
            Degrees of freedom        4
            p-value           0
            Chi-square statistic (baseline model)   211780
            Degrees of freedom (baseline model)     5
            CFI              0.996
            NFI              0.996
            NNFI             0.995
            RMSEA            0.49
Parameter Estimates
                      Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1              1.000
    close2              1.000
    close3              1.000
    close4              1.000
    close5              1.000
  F2 =~
    close1              0.000
    close2              2.000
    close3              3.000
    close4              4.000
    close5              5.000

Regressions:
  F1 ~
    gendr (b__F1)      -0.274    0.162   -1.686    0.092
  F2 ~
    gendr (b__F2)      -0.052    0.042   -1.224    0.221

Covariances:
  F1 ~~
    F2      (r_F1)      0.242    0.068    3.572    0.000

Intercepts:
    F1      (a_F1)     38.128    0.114  335.436    0.000
    F2      (a_F2)     -0.330    0.030  -11.102    0.000
    gender              0.490    0.017   28.560    0.000
    close1              0.000
    close2              0.000
    close3              0.000
    close4              0.000
    close5              0.000

Variances:
    F1      (e_F1)      3.055    0.314    9.742    0.000
    F2      (e_F2)      0.127    0.023    5.544    0.000
    close1 (e_c1)       3.527    0.289   12.219    0.000
    close2 (e_c2)       2.989    0.182   16.470    0.000
    close3 (e_c3)       4.004    0.235   17.056    0.000
    close4 (e_c4)       3.665    0.237   15.471    0.000
    close5 (e_c5)       4.509    0.318   14.194    0.000
    gender              0.250    0.012   20.616    0.000
```
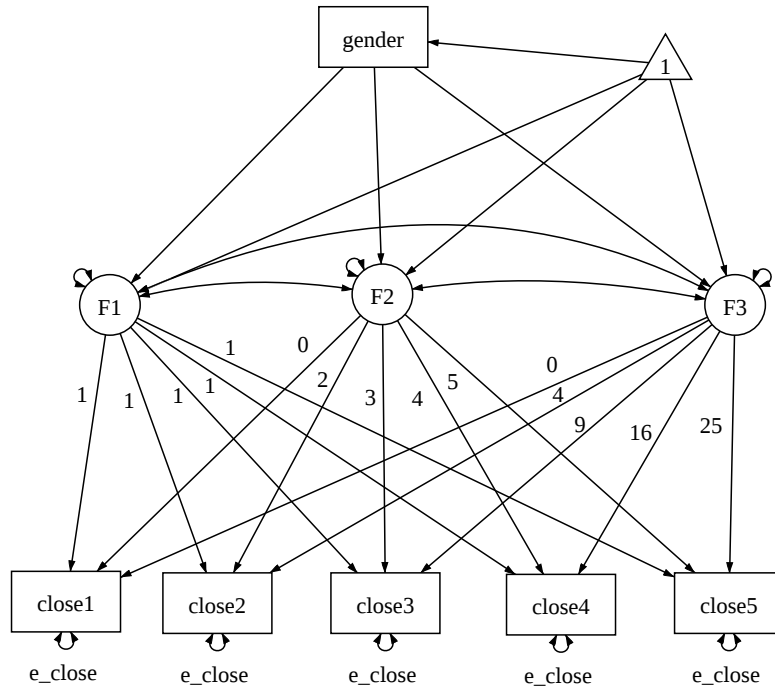
Figure 6: Quadratic growth curve model with equal variances and a covariate

4.3 The nonlinear growth curve model with equal variance in Figure 6 is then fitted to the data and the output is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    53.631
        Degrees of freedom      12
        p-value         0
        Chi-square statistic (baseline model)   9448.2
        Degrees of freedom (baseline model)     20
        CFI             0.996
        NFI             0.994
        NNFI            0.993
        RMSEA           0.064
Fit for covariance only
        Chi-square statistic    51.52
        Degrees of freedom      9
        p-value         0
        Chi-square statistic (baseline model)   1971.2
        Degrees of freedom (baseline model)     15
        CFI             0.978
        NFI             0.974
        NNFI            0.964
        RMSEA           0.075
Fit for mean only
```

```
          Chi-square statistic    818.76
          Degrees of freedom      3
          p-value          0
          Chi-square statistic (baseline model)   211202
          Degrees of freedom (baseline model)     5
          CFI               0.996
          NFI               0.996
          NNFI              0.994
          RMSEA             0.565
Parameter Estimates
                    Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    close1              1.000
    close2              1.000
    close3              1.000
    close4              1.000
    close5              1.000
  F2 =~
    close1              0.000
    close2              2.000
    close3              3.000
    close4              4.000
    close5              5.000
  F3 =~
    close1              0.000
    close2              4.000
    close3              9.000
    close4             16.000
    close5             25.000

Regressions:
  F1 ~
    gendr (b__F1)      -0.341     0.171    -2.001     0.045
  F2 ~
    gendr (b__F2)       0.036     0.120     0.296     0.767
  F3 ~
    gendr              -0.018     0.023    -0.760     0.447

Covariances:
  F1 ~~
    F2     (r_F1)       0.305     0.175     1.745     0.081
  F2 ~~
    F3                 -0.101     0.032    -3.140     0.002
  F1 ~~
    F3                 -0.014     0.032    -0.452     0.652

Intercepts:
    F1     (a_F1)      38.110     0.119   318.989     0.000
    F2     (a_F2)      -0.281     0.084    -3.343     0.001
    gender              0.490     0.017    28.560     0.000
    F3                 -0.011     0.016    -0.643     0.520
    close1              0.000
    close2              0.000
```
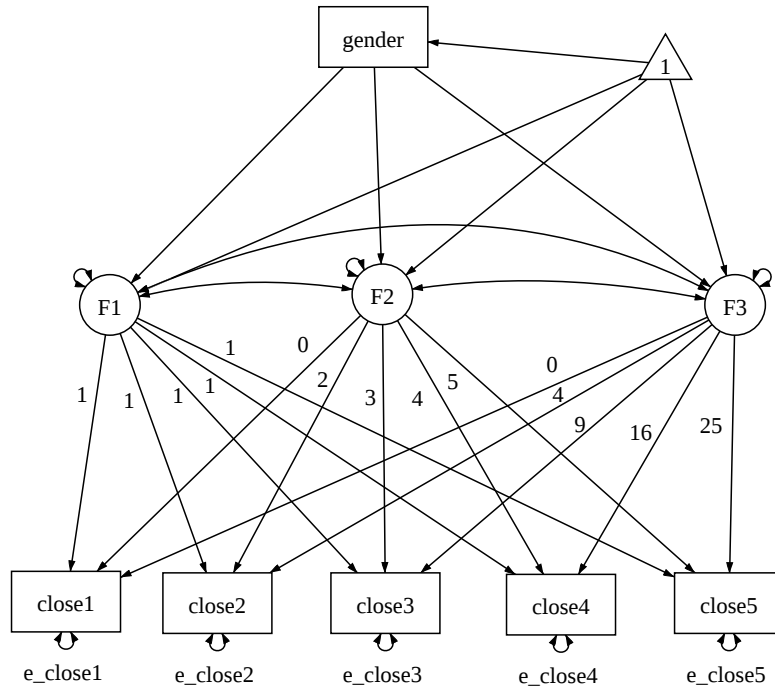
Figure 7: Quadratic growth curve model with unequal variances and a covariate

```
      close3              0.000
      close4              0.000
      close5              0.000

Variances:
      F1       (e_F1)     2.880     0.321      8.970     0.000
      F2       (e_F2)     0.587     0.171      3.426     0.001
      close1   (e_cl)     3.412     0.117     29.155     0.000
      close2   (e_cl)     3.412     0.117     29.155     0.000
      close3   (e_cl)     3.412     0.117     29.155     0.000
      close4   (e_cl)     3.412     0.117     29.155     0.000
      close5   (e_cl)     3.412     0.117     29.155     0.000
      F3                  0.023     0.006      3.578     0.000
      gender              0.250     0.012     20.616     0.000
```

4.4 Nonlinear model with unequal variances

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    26.958
        Degrees of freedom       8
        p-value           1e-03
        Chi-square statistic (baseline model)  9448.2
        Degrees of freedom (baseline model)     20
        CFI               0.998
```

```
             NFI               0.997
             NNFI              0.995
             RMSEA             0.053
Fit for covariance only
             Chi-square statistic   24.669
             Degrees of freedom      5
             p-value           0
             Chi-square statistic (baseline model)   1971.2
             Degrees of freedom (baseline model)     15
             CFI               0.99
             NFI               0.987
             NNFI              0.97
             RMSEA             0.068
Fit for mean only
             Chi-square statistic   818.94
             Degrees of freedom      3
             p-value           0
             Chi-square statistic (baseline model)   213725
             Degrees of freedom (baseline model)     5
             CFI               0.996
             NFI               0.996
             NNFI              0.994
             RMSEA             0.565
Parameter Estimates
                     Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1             1.000
    close2             1.000
    close3             1.000
    close4             1.000
    close5             1.000
  F2 =~
    close1             0.000
    close2             2.000
    close3             3.000
    close4             4.000
    close5             5.000
  F3 =~
    close1             0.000
    close2             4.000
    close3             9.000
    close4            16.000
    close5            25.000

Regressions:
  F1 ~
    gender            -0.319    0.170    -1.871    0.061
  F2 ~
    gender             0.053    0.120     0.443    0.658
  F3 ~
    gender            -0.023    0.023    -0.972    0.331

Covariances:
```

```
    F1 ~~
      F2      (r_F1)       1.162       0.472       2.463       0.014
    F2 ~~
      F3                  -0.052       0.049      -1.066       0.287
    F1 ~~
      F3                  -0.135       0.069      -1.946       0.052

Intercepts:
    F1                    38.092       0.119     319.590       0.000
    F2                    -0.295       0.084      -3.521       0.000
    gender                 0.490       0.017      28.560       0.000
    F3                    -0.007       0.016      -0.405       0.686
    close1                 0.000
    close2                 0.000
    close3                 0.000
    close4                 0.000
    close5                 0.000

Variances:
    F1       (e_F1)        1.546       0.764       2.023       0.043
    F2       (e_F2)        0.154       0.315       0.490       0.624
    close1   (e_c1)        4.839       0.773       6.259       0.000
    close2   (e_c2)        2.394       0.197      12.157       0.000
    close3   (e_c3)        3.723       0.232      16.023       0.000
    close4   (e_c4)        3.924       0.247      15.870       0.000
    close5   (e_c5)        3.243       0.451       7.195       0.000
    F3                     0.019       0.009       2.242       0.025
    gender                 0.250       0.012      20.616       0.000
```

1.5 Missing data and non-normal data

For the analysis of missing data and non-normal data, the raw data are needed. The data used in this example are given below.

```
piat1997 piat1998 piat1999 piat2000 income1 fgrade mgrade asvab
NA 65 50 61 NA 10 15 8.334
48 44 50 NA NA NA 8 NA
60 37 42 52 NA NA 8 NA
61 85 69 71 17.626 12 12 47.134
NA 38 72 69 8 NA 14 58.541
59 79 61 57 NA 14 14 52.921
52 72 79 69 NA NA 95 27.471
73 79 82 73 2.076 12 11 NA
41 67 NA NA NA 12 12 2.39
64 69 72 NA NA 12 12 NA
73 74 82 NA NA 12 12 NA
69 78 72 70 42 12 16 39.585
51 45 50 70 1 12 12 18.371
63 72 69 74 NA NA 11 12.233
80 85 89 92 23 NA 11 53.768
78 81 96 97 NA 12 14 56.63
74 50 NA 62 NA 12 11 NA
80 65 85 86 5.8 16 13 NA
63 72 64 NA NA 11 14 26.195
```

```
68 86 73 90 6 NA 15 27.912
82 92 95 97 NA 15 16 77.604
61 58 79 72 34 12 12 27.664
76 55 69 82 0.201 12 10 NA
80 81 87 85 50 16 18 51.582
50 48 61 73 2 12 9 16.523
60 55 62 NA 26 16 13 NA
76 82 NA NA NA 14 18 70.421
64 NA 57 65 NA 12 NA NA
93 93 95 97 80 16 16 97.811
50 58 77 76 NA NA NA 9.674
49 54 72 65 37 12 14 26.572
58 62 81 81 14 16 16 NA
60 70 72 82 10 NA NA 12.534
67 90 84 72 25 NA 12 35.74
68 80 85 90 50 12 9 22.08
66 59 60 70 40 12 14 2.167
75 76 81 85 12.978 NA NA 14.279
90 89 97 95 NA 12 16 NA
73 76 81 79 NA 7 16 29.276
44 50 NA 55 NA NA NA 0.835
31 NA 28 NA NA NA 11 12.321
60 56 67 65 0 12 12 17.119
NA NA 87 74 NA 16 9 46.48
79 68 98 88 26 11 12 30.329
92 80 91 NA NA 3 6 34.815
35 62 70 74 NA 18 13 15.719
39 50 71 NA NA NA 9 14.495
61 88 63 83 NA 12 15 94.837
70 63 64 70 13 14 12 NA
69 53 65 66 13 NA 11 2.675
52 54 49 NA NA NA 12 NA
60 67 75 70 0 NA 12 5.648
24 48 42 52 0.346 NA NA 10.056
40 50 66 67 NA NA 12 47.984
NA 29 49 49 NA NA 12 6.732
55 63 79 69 NA 16 16 76.56
60 47 52 49 3.2 14 15 28.346
54 64 78 68 3 NA 14 NA
61 75 77 93 NA 12 12 NA
58 47 61 50 51.5 12 15 77.556
40 56 51 62 12.48 NA NA NA
85 74 100 92 25 11 12 NA
60 57 83 77 17 NA 9 NA
58 NA 49 55 20 8 14 11.463
60 88 89 89 0 13 13 78.616
75 73 74 82 30 11 10 NA
54 45 43 57 NA 11 10 NA
44 NA 46 73 NA 12 12 71.217
30 57 30 59 NA NA 8 25.968
70 NA 94 NA 25 14 14 NA
51 59 49 25 NA 9 10 42.665
84 89 95 93 19 16 16 81.228
58 67 NA 83 0.25 18 18 62.397
```

```
61 60 65 89 NA 12 12 24.844
56 51 61 60 NA 12 12 37.476
65 68 72 66 11 NA 12 14.151
66 73 NA 69 13 17 18 27.771
58 58 60 74 10 13 14 49.108
39 53 NA NA NA NA 12 19.208
59 76 50 NA 0.8 15 12 20.767
60 NA 60 NA NA 12 12 NA
79 74 74 66 19 NA 10 23.715
62 74 72 NA NA 13 13 13.75
NA 66 49 66 NA 16 16 NA
85 88 91 97 16 12 13 45.173
68 NA 68 70 14.4 NA 11 4.148
58 49 54 39 NA 12 NA 1.523
50 62 50 74 10 12 11 NA
78 93 NA 93 54.56 12 12 18.284
58 75 61 52 8 4 6 0.578
63 59 83 67 14 NA 12 NA
67 83 90 92 21 12 9 30.856
79 78 86 94 10 12 12 NA
49 58 66 56 23 12 12 NA
50 70 64 68 174.605 12 14 57.45
48 60 56 52 NA 12 14 78.004
97 92 72 71 17 NA NA 97.332
70 NA 78 74 NA 15 14 17.147
67 74 NA NA 3 NA 12 4.378
41 65 66 60 32 15 12 NA
95 96 95 99 21 12 12 3.638
NA 86 88 93 23 NA 15 80.289
NA 60 67 NA NA 12 14 54.581
66 71 72 NA 26 16 16 68.893
74 NA 88 89 28 16 12 10.889
49 64 61 55 NA NA 12 NA
55 42 27 41 NA 16 16 90.539
63 50 52 69 6 16 16 11.417
NA 59 75 65 NA NA NA 57.619
42 71 NA NA 42.5 14 16 30.579
76 83 80 86 0.013 11 11 19.265
67 64 63 92 48 12 11 60.68
69 70 73 76 15 12 12 17.255
56 NA 43 68 NA 12 14 20.272
71 55 72 66 NA 12 12 36.293
60 37 88 78 42 10 12 47.193
83 73 NA NA 6 12 14 68.082
69 66 61 73 NA 16 NA 89.749
71 85 74 70 21 9 14 18.671
62 65 NA 22 18.9 NA 12 NA
53 65 67 73 30.5 13 NA 36.962
NA 41 34 NA NA NA 12 23.558
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52 62 70 60 20 NA 13 10.672
16 20 23 26 NA 10 13 71.841
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85 89 92 NA NA 10 12 0.297
66 70 66 69 NA NA 11 NA
72 73 74 60 30 16 16 92.602
68 82 79 77 46.6 NA 14 61.967
60 67 66 81 5 NA 11 3.172
43 62 NA 58 5 NA 11 9.832
55 47 34 65 NA NA 11 19.029
80 90 89 64 33 NA NA 59.867
67 87 99 87 0 12 13 68.917
79 88 73 92 0.03 12 12 30.257
81 78 82 87 23 NA 12 39.821
65 85 75 88 NA 12 12 8.281
28 2 54 61 NA 8 14 14.754
88 87 90 97 NA 12 12 1.778
75 76 83 87 21 16 18 NA
65 63 69 63 2 16 16 48.117
75 73 79 81 NA 12 12 75.756
70 72 85 89 8.5 6 11 43.971
58 46 53 70 19 NA 12 8.901
83 87 86 97 NA NA NA 43.183
66 69 68 89 14.556 12 12 28.106
65 59 66 73 1.5 12 12 78.796
37 36 NA 43 16 12 12 60.091
78 83 63 74 12.9 11 8 12.829
47 55 58 67 9.998 8 8 29.464
97 50 45 100 NA 12 14 NA
52 56 56 100 3 12 9 NA
44 23 39 100 NA NA 6 8.275
46 NA 49 NA NA 12 17 38.04
55 66 66 68 NA 12 11 4.429
47 52 63 66 NA NA 11 8.728
49 63 59 62 8 11 10 2.38
52 50 56 62 8 19 12 22.071
50 47 54 63 NA NA 11 1.162
52 66 NA 46 NA 15 14 29.126
56 69 50 75 NA 16 12 0.627
73 86 75 75 9.912 12 12 4.712
53 57 56 69 19 12 12 60.806
70 72 70 72 0 12 11 NA
57 55 55 53 NA 12 12 55.006
51 58 64 70 17 11 14 81.669
53 57 60 70 15.6 NA 11 30.261
50 70 70 72 18 13 12 32.471
56 55 65 70 NA NA 14 37.494
49 47 89 69 NA NA 10 21.307
65 72 65 76 NA 13 14 33.141
75 78 92 94 6.5 14 12 NA
68 71 56 75 NA 12 17 41.186
55 61 60 64 21 15 12 15.03
63 71 66 67 NA 13 12 77.568
56 50 0 59 1.257 10 9 9.566
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62 67 63 57 30 18 16 69.921
73 48 NA 45 NA 10 12 68.913
62 53 73 65 30 12 11 NA
56 53 51 NA NA 11 NA 65.828
92 70 71 75 NA 11 12 43.517
NA 59 65 64 13 12 14 62.24
71 69 69 75 50 NA 14 68.115
86 82 90 91 NA 16 NA 51.648
86 87 92 94 20 17 18 87.545
NA 36 41 46 NA 17 16 35.874
58 54 65 64 NA 11 11 NA
69 71 87 81 NA NA 12 NA
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50 70 68 80 5 5 6 NA
NA 43 NA 48 NA 12 12 NA
76 59 73 89 20 NA 6 80.12
75 90 92 97 1.813 12 NA 62.376
43 43 71 46 10 13 13 79.836
40 44 40 40 10 13 13 27.214
48 48 80 84 0 NA 12 16.169
70 73 91 71 13 14 15 20.729
46 45 49 63 NA 12 12 28.229
38 58 66 65 NA NA 14 NA
49 46 66 65 21 3 3 NA
74 NA 70 85 30 NA 10 NA
46 75 78 74 34 NA 6 33.373
70 71 87 91 10 12 15 6.285
65 40 60 70 0 NA NA NA
61 54 56 55 NA NA NA NA
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75 81 NA 93 3 16 12 45.046
35 55 57 34 NA 6 6 2.354
73 72 84 84 NA 16 9 15.469
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63 93 81 89 NA NA 11 19.016
73 79 71 75 40 NA 10 10.487
63 84 75 64 40 NA 11 46.134
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66 68 69 68 50 NA NA 1.239
61 57 38 43 0 NA 12 NA
63 53 90 85 NA NA NA 10.209
NA 54 NA 70 15 14 14 68.315
66 70 60 69 13.368 NA 12 5.745
73 94 92 95 24 NA NA 8.912
61 50 49 50 0 12 12 7.563
91 NA 94 NA 32 12 11 23.034
83 76 72 NA NA 9 9 3.123
51 57 NA NA NA NA NA 15.949
72 73 69 75 NA 11 8 NA
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53 69 NA 1 NA 6 8 2.27
64 75 88 NA NA 14 12 NA
24 NA NA 45 NA 12 12 31.063
```

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62 89 91 90 NA 12 9 23.724
56 65 69 4 2.675 13 12 18.997
63 60 69 64 21 12 8 70.42
49 67 77 90 3 12 12 5.226
68 71 71 79 23 14 20 49.089
59 68 64 62 NA 17 18 50.701
55 59 68 71 41 11 14 NA
80 81 76 99 43 NA 10 NA
48 46 45 66 NA 18 16 42.198
60 68 69 70 10.4 12 12 NA
57 64 68 68 34 NA 11 5.428
63 73 70 57 10.5 12 11 18.953
51 60 53 51 2.328 NA 10 25.593
60 74 69 83 NA 12 11 24.889
53 58 61 61 8 12 12 57.984
60 62 57 60 NA 8 10 12.671
66 75 75 NA 13 12 14 29.434
53 38 36 NA NA 12 16 NA
47 37 99 98 1.7 12 12 NA
60 60 94 NA NA NA 8 3.085
69 NA 71 87 174.605 NA 3 NA
48 23 26 41 5.928 14 12 NA
54 38 53 NA 5.928 16 14 19.598
55 59 57 74 18 5 3 NA
65 80 68 72 NA NA 5 7.784
38 54 44 52 NA 8 6 15.716
38 28 54 66 NA NA 12 25.548
17 40 8 22 0 17 13 NA
25 26 70 18 0 17 13 NA
25 35 64 55 28 NA 7 3.832
NA 23 41 28 NA 12 13 NA
NA 73 62 85 NA 16 16 24.729
48 44 59 53 9.907 12 13 39.893
58 70 69 61 9 NA 14 10.666
80 88 87 93 NA 12 12 0.95
64 66 86 86 30 12 10 11.229
23 1 NA 55 NA NA NA NA
65 68 64 69 48 12 NA 6.724
85 73 58 64 18 14 12 2.009
55 67 69 77 18 16 17 69.876
45 52 56 52 NA 14 9 NA
31 19 19 50 NA 12 12 19.437
72 88 NA 82 18 11 14 NA
20 23 38 38 4 12 13 NA
56 68 72 70 2 5 11 7.295
28 33 52 NA NA NA 8 2.079
64 78 80 74 NA 12 11 34.026
60 54 66 67 12 9 12 10.8
44 38 47 65 2 NA 12 8.322
60 61 63 60 NA 11 12 26.82
54 62 69 51 NA 12 12 16.078
```

```
49  47  68  62  NA  12  12  26.185
44  49  47  47  NA  NA  16  25.273
66  66  73  93  7.5  NA  11  NA
43  59  NA  NA  27.648  12  12  NA
63  73  72  88  30  NA  11  NA
65  77  93  NA  NA  NA  12  NA
92  93  97  97  16  8  8  18.304
60  75  83  69  10  12  NA  8.453
59  68  73  75  NA  12  12  4.971
83  96  88  93  13  NA  NA  22.839
20  61  58  21  NA  NA  NA  15.773
63  69  NA  NA  14  12  9  8.82
76  81  85  67  NA  20  15  52.743
45  54  53  49  NA  11  12  NA
73  49  56  42  22  14  13  15.466
NA  46  59  57  27  NA  9  1.148
69  82  NA  72  NA  12  12  3.725
65  69  NA  79  8  12  12  5.925
42  44  47  50  20  12  11  0.316
61  68  72  73  NA  NA  14  43.981
87  90  92  97  11  12  11  0.204
67  74  69  77  0.8  16  14  8.616
90  92  90  91  7  NA  NA  46.574
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NA  59  61  100  NA  NA  13  69.753
85  90  89  90  NA  NA  14  NA
70  69  64  78  83  12  12  46.931
59  75  69  70  18  12  12  NA
71  65  91  94  NA  NA  12  NA
76  80  73  75  22  NA  12  24.204
51  54  51  55  NA  15  16  26.696
69  NA  80  71  0  14  15  22.92
48  57  60  59  16  NA  12  9.64
54  41  59  68  5  12  12  22.39
58  67  94  95  10  11  8  24.39
54  48  58  70  NA  14  11  1.518
64  54  52  35  NA  NA  11  0
53  72  68  66  28.5  12  11  11.839
60  70  67  65  NA  10  11  10.045
31  NA  40  46  2  12  14  1.141
59  61  69  NA  NA  11  11  10.267
59  50  100  61  NA  12  11  2.007
67  76  74  75  15  14  13  NA
62  92  NA  NA  NA  NA  12  27.468
52  66  92  76  10  11  12  83.985
42  6  48  75  NA  7  8  NA
54  77  90  60  20  NA  11  NA
81  73  NA  NA  46  14  12  68.089
34  36  44  54  NA  NA  15  7.855
66  70  NA  NA  NA  12  10  23.616
86  56  70  NA  NA  3  5  NA
66  72  79  NA  NA  14  12  NA
49  73  70  78  0  NA  12  NA
62  50  61  81  NA  14  12  58.405
```

```
54 NA 64 64 NA 14 12 NA
93 90 97 98 27 11 10 10.541
22 20 NA 27 14 12 9 15.867
70 70 79 83 22 NA 14 46.848
81 NA NA 91 25 12 16 95.621
58 73 73 97 23 12 12 11.697
100 5 32 39 NA NA 12 38.443
95 98 97 97 16 NA 10 28.367
54 60 60 70 18.95 NA 12 NA
72 70 NA NA 5 13 10 21.795
100 49 61 56 1.7 12 12 NA
100 82 72 95 0 12 12 35.54
100 71 87 67 15 12 14 16.219
65 70 NA 50 NA 13 12 29.185
68 56 88 80 55 14 14 71.297
48 NA NA 71 NA 6 12 5.353
55 88 76 94 41 NA 12 1.535
78 77 88 88 15 4 9 22.874
86 91 91 88 15 NA NA 11.974
62 69 NA 49 NA NA 10 10.403
31 45 50 41 NA 12 11 21.898
46 64 NA 73 24 NA 13 NA
61 67 64 65 0.9 12 15 68.3
81 58 52 NA 17.056 NA NA 4.91
59 56 59 75 NA NA 15 NA
NA 66 57 71 NA 8 7 11.555
29 44 44 42 NA 12 12 29.784
49 52 54 62 14 12 12 22.651
43 78 70 90 17 10 12 44.454
83 68 76 71 27 NA 12 62.57
63 65 79 72 NA NA 12 15.447
55 59 59 58 NA NA 11 2.757
51 69 52 60 9 NA 10 NA
28 61 NA 59 NA 11 12 NA
44 41 75 83 15 12 11 65.985
58 56 67 66 15 12 11 NA
54 47 58 59 5 6 12 6.615
NA 40 55 68 NA NA NA 1.867
46 61 54 70 19 NA NA 3.256
44 56 66 54 3 3 NA NA
62 68 NA 81 10 16 12 NA
63 69 66 68 NA 15 12 9.809
61 59 61 64 7.2 NA 7 54.275
46 54 64 49 NA NA 12 3.302
NA 69 78 58 25 7 12 51.4
NA 57 60 74 25 NA 12 12.546
68 37 51 56 11 12 6 14.316
74 64 78 80 47 16 16 22.618
70 65 65 59 18 NA 6 54.423
83 73 91 93 65 12 12 3.654
74 70 53 64 1 9 12 13.816
59 56 80 57 20 12 NA NA
72 NA NA 49 NA 14 14 60.778
78 91 89 87 43 12 12 5.262
```

```
31 1 47 49 0.003 12 12 43.201
84 77 65 87 0.04 10 13 9.527
68 70 79 NA 8 NA 12 NA
77 NA 56 55 NA 3 11 NA
63 0 64 71 0.028 9 12 NA
42 50 57 57 NA NA 10 2.011
31 37 57 41 NA 11 11 45.069
54 71 88 88 12 12 NA 16.714
54 NA 53 46 NA 12 14 44.382
78 NA NA 35 NA 12 13 34.472
60 62 62 61 NA 4 9 NA
81 61 72 80 NA 12 12 75.4
88 90 70 92 25 9 9 45.651
61 63 63 60 NA 16 16 NA
43 43 54 48 NA 19 12 79.208
24 30 27 27 2.2 2 3 2.575
63 58 64 51 15.3 12 12 NA
79 80 82 87 26 12 14 7.54
46 48 63 68 7 NA 2 NA
57 55 50 65 4 NA NA 6.33
66 69 76 69 NA 12 15 14.116
68 69 68 70 8.736 NA NA NA
59 60 64 61 15 NA NA NA
73 86 88 90 NA NA NA NA
69 65 69 74 NA NA NA NA
```

The following model is fitted to the data. Note that in the weight field, 0.1 (0 can also be used not to down-weight any data) is input to specify how much data to down-weight. In the control part, we add "mean.fit" to request the fit for mean and covariance.

The output is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic   13.836
        Degrees of freedom      5
        p-value        0.017
        Chi-square statistic (baseline model)  1106.8
        Degrees of freedom (baseline model)    9
        CFI            0.994
        NFI            0.987
        NNFI           0.986
        RMSEA          0.058
Fit for covariance only
        Chi-square statistic   11.907
        Degrees of freedom      3
        p-value        8e-03
        Chi-square statistic (baseline model)  1024.7
        Degrees of freedom (baseline model)    6
        CFI            0.992
        NFI            0.988
        NNFI           0.983
        RMSEA          0.078
Fit for mean only
        Chi-square statistic   1.934
        Degrees of freedom      2
```
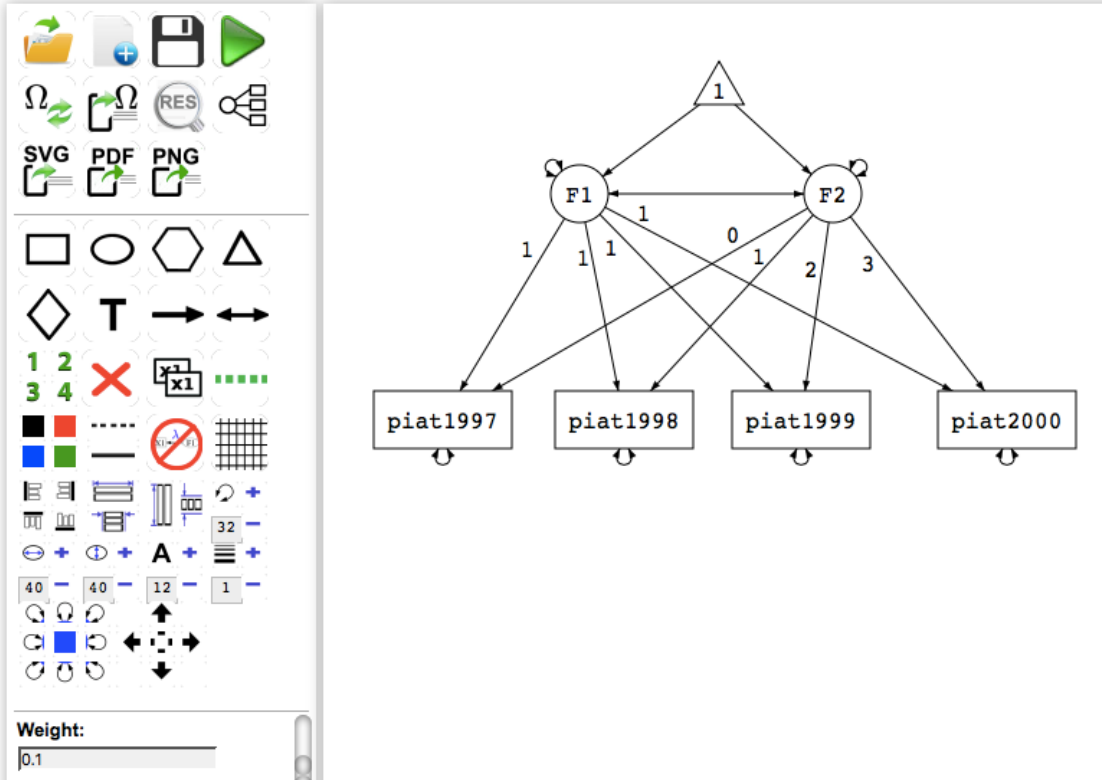
Figure 8: Path diagram for missing data and robust method

```
        p-value             0.38
        Chi-square statistic (baseline model)   216.56
        Degrees of freedom (baseline model)     3
        CFI             1
        NFI             0.991
        NNFI            1
        RMSEA           0
Parameter Estimates
                    Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    piat1997            1.000
    piat1998            1.000
    piat1999            1.000
    piat2000            1.000
  F2 =~
    piat1997            0.000
    piat1998            1.000
    piat1999            2.000
    piat2000            3.000

Covariances:
  F1 ~~
    F2                  -5.060     3.954    -1.279     0.201

Intercepts:
```

```
        F2                    3.100      0.212    14.633    0.000
        F1                   61.010      0.716    85.264    0.000
        piat1997              0.000
        piat1998              0.000
        piat1999              0.000
        piat2000              0.000

 Variances:
        F1                  170.729     15.570    10.965    0.000
        F2                    6.264      1.840     3.404    0.001
        piat1997             61.772      8.769     7.044    0.000
        piat1998             71.972      6.470    11.124    0.000
        piat1999             67.611      6.306    10.721    0.000
        piat2000             63.244      9.028     7.005    0.000
```