

# Supplementary Materials for Yuan, Zhang, and Deng

September 14, 2015

## 1 Using WebSEM for Data Analysis

The fit indices for mean structures with growth curve models developed in this article are currently available in WebSEM, which is free online software that allows researchers to conduct SEM analysis through path diagrams (Zhang & Yuan, 2012-2015).<sup>1</sup> In this section, we illustrate how to obtain the fit indices in WebSEM through the linear model with equal error variances in Example 4 used in the paper. A video with instructions on how to conduct the data analysis is also available on YouTube (<https://youtu.be/1ydG3IxBIsm>).

To use WebSEM, one needs to provide a data file and draw a path diagram. The data file should be a free format text file with the extension name txt. The data file can be generated using any text editor or the default editor within WebSEM. Both raw data and summary data can be used. If raw data are used, the first line of the data file should be variable names and the rest are data entries. If summary data are used, the first row includes the variable names. The second row provides the value of the sample size that is repeated for each column of data. The third row includes information on the sample means. The rest is the full sample covariance matrix. For example, the summary data in the data file for Example 4 are given below. Clearly, there are six variables named close1, close2, close3, close4, close5 and gender, respectively, and the sample size is 851. Note that the value 851 repeats itself 6 times.

---

close1	close2	close3	close4	close5	gender
851	851	851	851	851	851
37.9542	37.2785	37.0463	36.5696	36.1363	0.4900
6.3944	3.2716	4.1435	3.7058	4.1286	-0.0940
3.2716	7.5282	6.0804	5.1597	5.7608	-0.0390
4.1435	6.0804	10.7290	6.5672	7.2365	-0.1521
3.7058	5.1597	6.5672	10.2920	7.6463	-0.1104
4.1286	5.7608	7.2365	7.6463	12.9085	-0.1469
-0.0940	-0.0390	-0.1521	-0.1104	-0.1469	0.2502

---

WebSEM provides an intuitive and interactive interface to draw path diagram for a model for data analysis. Information on how to use the interface can be found in Zhang and Yuan (2012-2015). The path diagram for the linear growth curve model drawn by WebSEM is given in Figure 2a. The video on YouTube showed how to draw a path diagram interactively. Because the use of summary data instead of raw data, we put “`data=cov`” in the control field of WebSEM. Furthermore, to request the fit indices for the mean structure, type “`mean.fit`” in the control field; otherwise, only fit indices for the combined mean and covariance structure will be produced.

The output of WebSEM by running the analysis is shown below.<sup>2</sup> Test Statistics and Fit Indices for both mean and covariance, covariance only, and mean only are given at the beginning of the output. For example, for evaluating both mean and covariance together, the chi-square statistic is 81.653 with 17 degrees of freedom. The corresponding p-value is essentially 0. The CFI is 0.971, NFI is 0.964, NNFI is 0.961 and RMSEA is 0.067. For mean only, the chi-square statistics is 5.073 (p-value = 0.167) and RMSEA is 0.028.

<sup>1</sup>Zhang, Z. & Yuan, K.-H. (2012-2015). WebSEM: Structural equation modeling online [Manual available at <http://psychstat.org/webssem>]. Retrievable from <https://websem.psychstat.org>.

<sup>2</sup>The known parameters are removed from the output to save space.

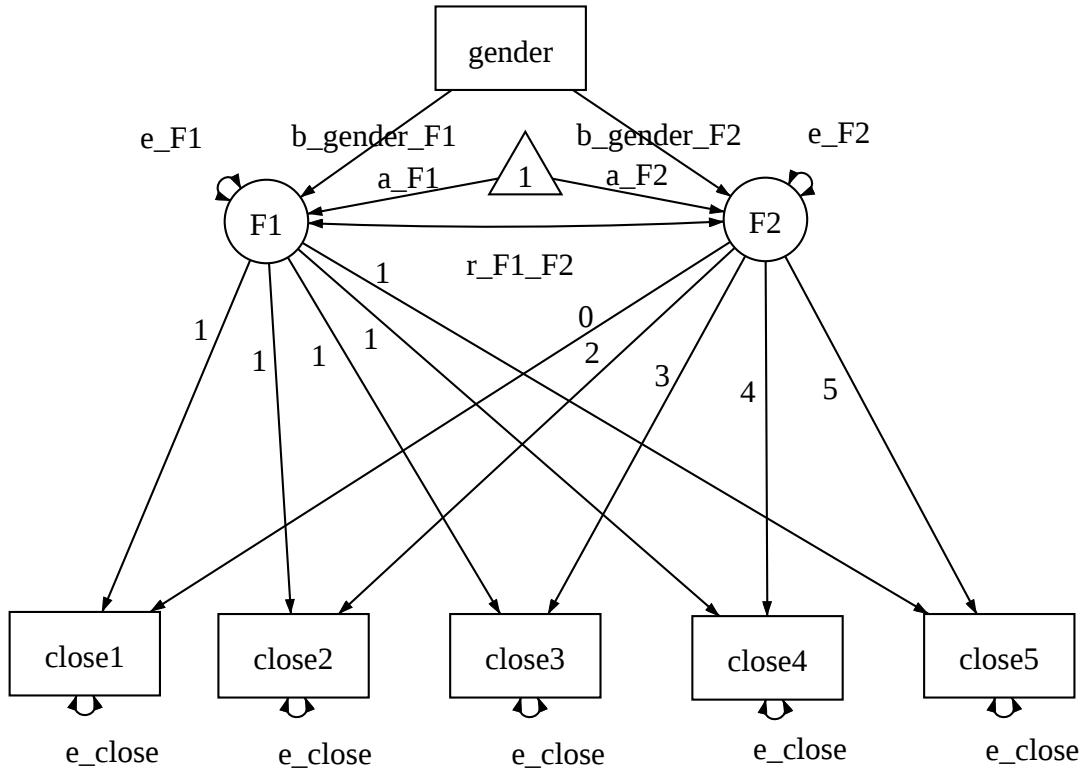


Figure 1: Path diagram for the linear growth curve model with equal error variances. The path diagram was drawn by WebSEM.

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Test Statistics and Fit Indices  
 Fit for both mean and covariance  
     Chi-square statistic   81.653  
     Degrees of freedom     17  
     p-value                0  
     Chi-square statistic (baseline model) 2272.4  
     Degrees of freedom (baseline model) 23  
     CFI                    0.971  
     NFI                    0.964  
     NNFI                  0.961  
     RMSEA                 0.067  
 Fit for covariance only  
     Chi-square statistic   76.586  
     Degrees of freedom     14  
     p-value                0  
     Chi-square statistic (baseline model) 1971.2  
     Degrees of freedom (baseline model) 19  
     CFI                    0.968  
     NFI                    0.961  
     NNFI                  0.956  
     RMSEA                 0.072  
 Fit for mean only  
     Chi-square statistic   5.073  
     Degrees of freedom     3  
     p-value                0.167  
     Chi-square statistic (baseline model) 287.88  
     Degrees of freedom (baseline model) 4  
     CFI                    0.993  
     NFI                    0.982  
     NNFI                  0.99  
     RMSEA                 0.028  
 Parameter Estimates  

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
F1 =~				
close1	1.000			
close2	1.000			
close3	1.000			
close4	1.000			
close5	1.000			
F2 =~				
close1	0.000			
close2	2.000			
close3	3.000			
close4	4.000			
close5	5.000			
Regressions:				
F1 ~				
gendr (b__F1)	-0.289	0.163	-1.770	0.077
F2 ~				
gendr (b__F2)	-0.052	0.043	-1.216	0.224

```

Covariances:
F1 ~~
F2      (r_F1)      0.249      0.056      4.420      0.000

Intercepts:
F1      (a_F1)      38.141      0.114      334.196      0.000
F2      (a_F2)      -0.333      0.030     -11.169      0.000
close1      0.000
close2      0.000
close3      0.000
close4      0.000
close5      0.000

Variances:
F1      (e_F1)      2.956      0.284      10.399      0.000
F2      (e_F2)      0.136      0.020      6.804      0.000
close1 (e_c1)      3.696      0.103      35.728      0.000
close2 (e_c1)      3.696      0.103      35.728      0.000
close3 (e_c1)      3.696      0.103      35.728      0.000
close4 (e_c1)      3.696      0.103      35.728      0.000
close5 (e_c1)      3.696      0.103      35.728      0.000

```

---

Path diagrams and complete output for all models used in the paper are available as supplementary materials at <http://>.

## 1.1 Example 1

The summary data used in this example are given below. The model in Figure 2a is used.

y1	y2	y3	y4	y5	y6
200	200	200	200	200	200
1.0	1.5	2.0	2.5	3.0	3.5
2.0	2.0	2.0	2.5	3.0	3.5
2.0	4.0	4.5	6.0	7.5	9.0
2.0	4.5	8.0	10.0	12.0	14.5
2.5	6.0	10.0	14.0	16.5	20.0
3.0	7.5	12.0	16.5	22.0	26.0
3.5	9.0	14.5	20.0	26.0	32.0

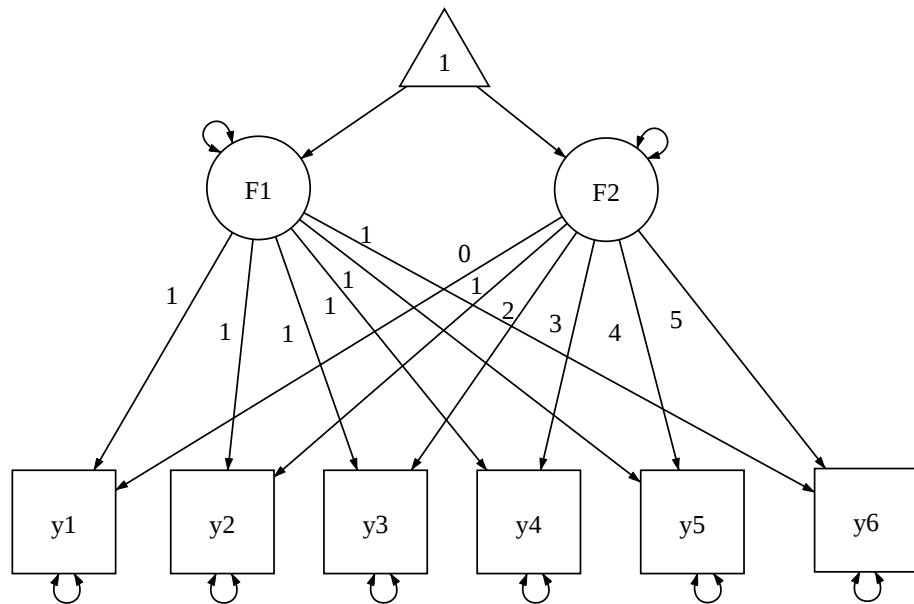
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The output of WebSEM by running the analysis is shown below.

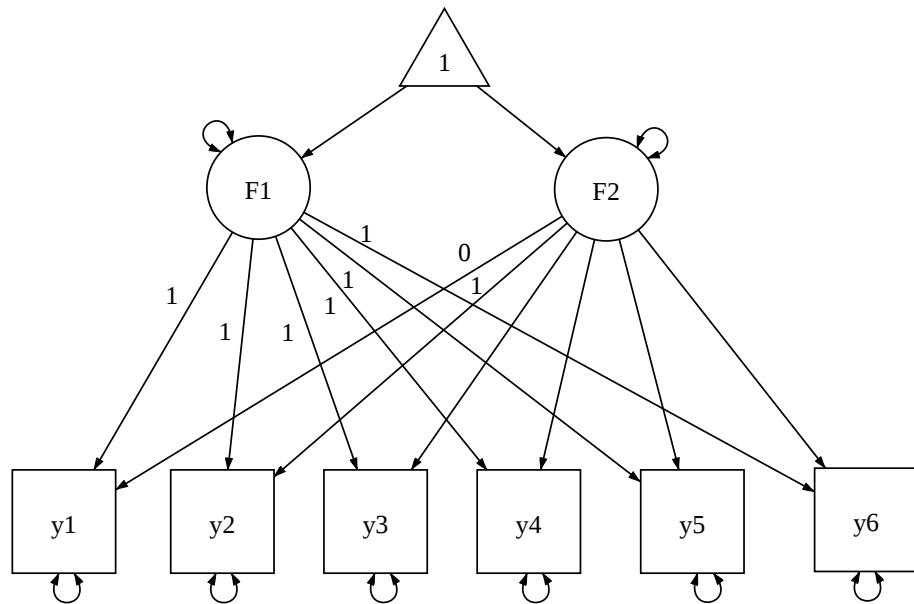
```

Test Statistics and Fit Indices
Fit for both mean and covariance
  Chi-square statistic    82.686
  Degrees of freedom      16
  p-value                  0
  Chi-square statistic (baseline model) 2010.6
  Degrees of freedom (baseline model)   20
  CFI                      0.966
  NFI                      0.959
  NNFI                     0.958
  RMSEA                    0.144
Fit for covariance only
  Chi-square statistic    82.686
  Degrees of freedom      12
  p-value                  0

```



(a) A linear growth curve model



(b) A nonlinear growth curve model

Figure 2: Path diagrams for models used in Examples 1-4

```

Chi-square statistic (baseline model) 1923.8
Degrees of freedom (baseline model) 15
CFI 0.963
NFI 0.957
NNFI 0.954
RMSEA 0.172

Fit for mean only
Chi-square statistic 0
Degrees of freedom 4
p-value 1
Chi-square statistic (baseline model) 46.505
Degrees of freedom (baseline model) 5
CFI 1
NFI 1
NNFI 1.12
RMSEA 0

Parameter Estimates
Estimate Std.error Z-value P(>|z|)

Latent variables:
F1 =~
y1 1.000
y2 1.000
y3 1.000
y4 1.000
y5 1.000
F2 =~
y1 0.000
y2 1.000
y3 2.000
y4 3.000
y5 4.000
F1 =~
y6 1.000
F2 =~
y6 5.000

Covariances:
F1 ~~
F2 (r_F1) 0.378 0.100 3.779 0.000

Intercepts:
F2 0.500 0.073 6.819 0.000
F1 1.000 0.094 10.667 0.000
y1 0.000
y2 0.000
y3 0.000
y4 0.000
y5 0.000
y6 0.000

Variances:
y1 0.532 0.103 5.186 0.000
y2 0.642 0.085 7.590 0.000
y3 1.040 0.119 8.740 0.000

```

---

y4	1.087	0.125	8.715	0.000
y5	0.595	0.090	6.627	0.000
F1	1.444	0.181	7.970	0.000
F2	1.044	0.108	9.710	0.000
y6	0.483	0.127	3.807	0.000

---

## 1.2 Example 2

The summary data used in this example are given below. The same model in Figure 2a is used.

---

y1	y2	y3	y4	y5	y6
200.0	200.0	200.0	200.0	200.0	200.0
1.0	1.5	2.0	2.5	3.0	3.5
2.0	2.2	2.0	2.5	3.0	3.5
2.2	4.0	4.5	6.0	7.5	9.0
2.0	4.5	8.0	10.2	12.0	14.5
2.5	6.0	10.2	14.0	16.5	20.0
3.0	7.5	12.0	16.5	22.0	26.2
3.5	9.0	14.5	20.0	26.2	32.0

---

The output of the analysis is given below.

---

```

Test Statistics and Fit Indices
Fit for both mean and covariance
    Chi-square statistic      188.53
    Degrees of freedom        16
    p-value                   0
    Chi-square statistic (baseline model) 2255.5
    Degrees of freedom (baseline model) 20
    CFI                      0.923
    NFI                      0.916
    NNFI                     0.904
    RMSEA                    0.232

Fit for covariance only
    Chi-square statistic      188.53
    Degrees of freedom        12
    p-value                   0
    Chi-square statistic (baseline model) 2168.6
    Degrees of freedom (baseline model) 15
    CFI                      0.918
    NFI                      0.913
    NNFI                     0.898
    RMSEA                    0.271

Fit for mean only
    Chi-square statistic      0
    Degrees of freedom        4
    p-value                   1
    Chi-square statistic (baseline model) 46.193
    Degrees of freedom (baseline model) 5
    CFI                      1
    NFI                      1
    NNFI                     1.121
    RMSEA                    0

Parameter Estimates
          Estimate   Std. err   Z-value   P(>|z|)
```

---

```

Latent variables:
F1 = ~
  y1          1.000
  y2          1.000
  y3          1.000
  y4          1.000
  y5          1.000
F2 = ~
  y1          0.000
  y2          1.000
  y3          2.000
  y4          3.000
  y5          4.000
F1 = ~
  y6          1.000
F2 = ~
  y6          5.000

Covariances:
F1 ~~
  F2      (r_F1)    0.326    0.103    3.160    0.002

Intercepts:
  F2          0.500    0.074    6.797    0.000
  F1          1.000    0.097   10.321    0.000
  y1          0.000
  y2          0.000
  y3          0.000
  y4          0.000
  y5          0.000
  y6          0.000

Variances:
  y1          0.288    0.073    3.957    0.000
  y2          0.438    0.058    7.525    0.000
  y3          1.188    0.130    9.115    0.000
  y4          1.226    0.134    9.180    0.000
  y5          0.405    0.064    6.311    0.000
  F1          1.687    0.192    8.793    0.000
  F2          1.065    0.108    9.838    0.000
  y6          0.255    0.093    2.751    0.006

```

---

### 1.3 Example 3

The summary data for Example 3 are given below.

y1	y2	y3	y4	y5	y6
200.0	200.0	200.0	200.0	200.0	200.0
1.2	1.5	2.0	2.5	3.0	3.2
2.0	1.5	2.0	2.5	3.0	3.5
1.5	4.0	4.5	6.0	7.5	9.0
2.0	4.5	8.0	9.5	12.0	14.5
2.5	6.0	9.5	14.0	16.5	20.0
3.0	7.5	12.0	16.5	22.0	25.5

3.5 9.0 14.5 20.0 25.5 32.0

---

The linear growth curve model in Figure 2a is first fitted to the data and the output is given below.

---

```
Test Statistics and Fit Indices
Fit for both mean and covariance
    Chi-square statistic    7.733
    Degrees of freedom      16
    p-value                 0.956
    Chi-square statistic (baseline model) 1716.8
    Degrees of freedom (baseline model)   20
    CFI                    1
    NFI                    0.995
    NNFI                   1.006
    RMSEA                  0

Fit for covariance only
    Chi-square statistic    0.075
    Degrees of freedom      12
    p-value                 1
    Chi-square statistic (baseline model) 1653
    Degrees of freedom (baseline model)   15
    CFI                    1
    NFI                    1
    NNFI                   1.009
    RMSEA                  0

Fit for mean only
    Chi-square statistic    7.697
    Degrees of freedom      4
    p-value                 0.103
    Chi-square statistic (baseline model) 42.64
    Degrees of freedom (baseline model)   5
    CFI                    0.902
    NFI                    0.819
    NNFI                   0.877
    RMSEA                  0.068

Parameter Estimates
                               Estimate  Std.err  Z-value  P(>|z| )
Latent variables:
  F1 =~
    y1                  1.000
    y2                  1.000
    y3                  1.000
    y4                  1.000
    y5                  1.000
  F2 =~
    y1                  0.000
    y2                  1.000
    y3                  2.000
    y4                  3.000
    y5                  4.000
  F1 =~
    y6                  1.000
  F2 =~
    y6                  5.000
```

Covariances:				
F1	~~			
F2		0.499	0.094	5.327
Intercepts:				
F2		0.429	0.073	5.902
F1		1.162	0.087	13.304
y1		0.000		
y2		0.000		
y3		0.000		
y4		0.000		
y5		0.000		
y6		0.000		
Variances:				
y1		0.999	0.146	6.819
y2		1.007	0.122	8.223
y3		0.994	0.116	8.599
y4		0.995	0.122	8.127
y5		1.018	0.151	6.761
F1		0.991	0.158	6.266
F2		0.994	0.105	9.440
y6		1.024	0.204	5.015

---

The nonlinear growth curve model in Figure 2b is then fitted to the data and the output is given below.

---

Test Statistics and Fit Indices				
Fit for both mean and covariance				
Chi-square statistic		6.429		
Degrees of freedom		12		
p-value		0.893		
Chi-square statistic (baseline model)		1716.8		
Degrees of freedom (baseline model)		20		
CFI		1		
NFI		0.996		
NNFI		1.005		
RMSEA		0		
Fit for covariance only				
Chi-square statistic		1.14		
Degrees of freedom		8		
p-value		0.997		
Chi-square statistic (baseline model)		1653		
Degrees of freedom (baseline model)		15		
CFI		1		
NFI		0.999		
NNFI		1.008		
RMSEA		0		
Fit for mean only				
Chi-square statistic		5.315		
Degrees of freedom		4		
p-value		0.256		
Chi-square statistic (baseline model)		40.555		
Degrees of freedom (baseline model)		5		
CFI		0.963		
NFI		0.869		

NNFI	0.954			
RMSEA	0.041			
Parameter Estimates				
	Estimate Std. err Z-value P(> z )			
Latent variables:				
F1 = ~				
y1	1.000			
y2	1.000			
y3	1.000			
y4	1.000			
y5	1.000			
F2 = ~				
y1	0.000			
y2	1.000			
y3	2.079	0.178	11.705	0.000
y4	3.159	0.276	11.435	0.000
y5	4.239	0.380	11.168	0.000
F1 = ~				
y6	1.000			
F2 = ~				
y6	5.193	0.472	10.993	0.000
Covariances:				
F1 ~~				
F2	0.490	0.092	5.313	0.000
Intercepts:				
F2	0.410	0.080	5.143	0.000
F1	1.165	0.090	12.987	0.000
y1	0.000			
y2	0.000			
y3	0.000			
y4	0.000			
y5	0.000			
y6	0.000			
Variances:				
y1	1.003	0.147	6.827	0.000
y2	1.002	0.122	8.180	0.000
y3	0.994	0.116	8.600	0.000
y4	0.990	0.122	8.085	0.000
y5	0.989	0.152	6.490	0.000
F1	1.005	0.165	6.081	0.000
F2	0.899	0.200	4.493	0.000
y6	1.064	0.208	5.115	0.000

## 1.4 Example 4

The summary data for Example 4 are given below.

y1	y2	y3	y4	y5	y6
200.0	200.0	200.0	200.0	200.0	200.0
1.2	1.5	2.0	2.5	3.0	3.1
2.0	1.5	2.0	2.5	3.0	3.5

1.5	4.0	4.5	6.0	7.5	9.0
2.0	4.5	8.0	9.5	12.0	14.5
2.5	6.0	9.5	14.0	16.5	20.0
3.0	7.5	12.0	16.5	22.0	25.5
3.5	9.0	14.5	20.0	25.5	32.0

The linear growth curve model in Figure 2a is first fitted to the data and the output is given below.

---

```

Test Statistics and Fit Indices
Fit for both mean and covariance
    Chi-square statistic      12.701
    Degrees of freedom        16
    p-value                   0.695
    Chi-square statistic (baseline model) 1715
    Degrees of freedom (baseline model)   20
    CFI                      1
    NFI                      0.993
    NNFI                     1.002
    RMSEA                    0

Fit for covariance only
    Chi-square statistic     0.244
    Degrees of freedom       12
    p-value                  1
    Chi-square statistic (baseline model) 1653
    Degrees of freedom (baseline model)   15
    CFI                      1
    NFI                      1
    NNFI                     1.009
    RMSEA                    0

Fit for mean only
    Chi-square statistic     12.519
    Degrees of freedom       4
    p-value                  0.014
    Chi-square statistic (baseline model) 45.286
    Degrees of freedom (baseline model)   5
    CFI                      0.789
    NFI                      0.724
    NNFI                     0.736
    RMSEA                    0.103

Parameter Estimates
                               Estimate  Std. err  Z-value  P(>|z| )
Latent variables:
  F1 = ~
    y1                  1.000
    y2                  1.000
    y3                  1.000
    y4                  1.000
    y5                  1.000
  F2 = ~
    y1                  0.000
    y2                  1.000
    y3                  2.000
    y4                  3.000
    y5                  4.000
  F1 = ~

```

y6	1.000			
F2 = ~				
y6	5.000			
<b>Covariances:</b>				
F1 ~~				
F2	0.500	0.094	5.336	0.000
<b>Intercepts:</b>				
F2	0.415	0.073	5.708	0.000
F1	1.180	0.087	13.497	0.000
y1	0.000			
y2	0.000			
y3	0.000			
y4	0.000			
y5	0.000			
y6	0.000			
<b>Variances:</b>				
y1	0.997	0.146	6.807	0.000
y2	1.009	0.123	8.227	0.000
y3	0.992	0.116	8.588	0.000
y4	0.995	0.123	8.098	0.000
y5	1.029	0.153	6.743	0.000
F1	0.990	0.158	6.254	0.000
F2	0.993	0.105	9.432	0.000
y6	1.062	0.209	5.081	0.000

The nonlinear growth curve model in Figure 2b is then fitted to the data and the output is given below.

<b>Test Statistics and Fit Indices</b>				
<b>Fit for both mean and covariance</b>				
Chi-square statistic	10.609			
Degrees of freedom	12			
p-value	0.563			
Chi-square statistic (baseline model)	1715			
Degrees of freedom (baseline model)	20			
CFI	1			
NFI	0.994			
NNFI	1.001			
RMSA	0			
<b>Fit for covariance only</b>				
Chi-square statistic	1.928			
Degrees of freedom	8			
p-value	0.983			
Chi-square statistic (baseline model)	1653			
Degrees of freedom (baseline model)	15			
CFI	1			
NFI	0.999			
NNFI	1.007			
RMSA	0			
<b>Fit for mean only</b>				
Chi-square statistic	8.725			
Degrees of freedom	4			
p-value	0.068			

Chi-square statistic (baseline model) 41.978  
 Degrees of freedom (baseline model) 5  
 CFI 0.872  
 NFI 0.792  
 NNFI 0.84  
 RMSEA 0.077

**Parameter Estimates**

	Estimate	Std.err	Z-value	P(> z )
<b>Latent variables:</b>				
F1 = ~				
y1	1.000			
y2	1.000			
y3	1.000			
y4	1.000			
y5	1.000			
F2 = ~				
y1	0.000			
y2	1.000			
y3	2.077	0.177	11.720	0.000
y4	3.155	0.275	11.452	0.000
y5	4.232	0.378	11.185	0.000
F1 = ~				
y6	1.000			
F2 = ~				
y6	5.144	0.467	11.012	0.000
<b>Covariances:</b>				
F1 ~~				
F2	0.487	0.092	5.270	0.000
<b>Intercepts:</b>				
F2	0.401	0.080	5.030	0.000
F1	1.178	0.090	13.147	0.000
y1	0.000			
y2	0.000			
y3	0.000			
y4	0.000			
y5	0.000			
y6	0.000			
<b>Variances:</b>				
y1	1.000	0.147	6.797	0.000
y2	1.003	0.123	8.185	0.000
y3	0.993	0.116	8.590	0.000
y4	0.989	0.123	8.045	0.000
y5	0.990	0.154	6.424	0.000
F1	0.997	0.165	6.048	0.000
F2	0.910	0.202	4.499	0.000
y6	1.114	0.211	5.268	0.000
Fit for both mean and covariance				Test Statistics and Fit Indicators
Chi-square statistic	10.609			
Degrees of freedom	12			
p-value	0.563			
Chi-square statistic (baseline model)	1715			

Degrees of freedom (baseline model) 20  
 CFI 1  
 NFI 0.994  
 NNFI 1.001  
 RMSEA 0  
 Fit for covariance only  
 Chi-square statistic 1.928  
 Degrees of freedom 8  
 p-value 0.983  
 Chi-square statistic (baseline model) 1653  
 Degrees of freedom (baseline model) 15  
 CFI 1  
 NFI 0.999  
 NNFI 1.007  
 RMSEA 0  
 Fit for mean only  
 Chi-square statistic 8.725  
 Degrees of freedom 4  
 p-value 0.068  
 Chi-square statistic (baseline model) 41.978  
 Degrees of freedom (baseline model) 5  
 CFI 0.872  
 NFI 0.792  
 NNFI 0.84  
 RMSEA 0.077  
**Parameter Estimates**  

	Estimate	Std. err	Z-value	P(> z )
<b>Latent variables:</b>				
F1 = ~				
y1	1.000			
y2	1.000			
y3	1.000			
y4	1.000			
y5	1.000			
F2 = ~				
y1	0.000			
y2	1.000			
y3	2.077	0.177	11.720	0.000
y4	3.155	0.275	11.452	0.000
y5	4.232	0.378	11.185	0.000
F1 = ~				
y6	1.000			
F2 = ~				
y6	5.144	0.467	11.012	0.000
<b>Covariances:</b>				
F1 ~~				
F2	0.487	0.092	5.270	0.000
<b>Intercepts:</b>				
F2	0.401	0.080	5.030	0.000
F1	1.178	0.090	13.147	0.000
y1	0.000			
y2	0.000			

y3	0.000
y4	0.000
y5	0.000
y6	0.000
<b>Variances:</b>	
y1	1.000
y2	1.003
y3	0.993
y4	0.989
y5	0.990
F1	0.997
F2	0.910
y6	1.114
	0.147
	8.185
	8.590
	8.045
	6.424
	6.048
	4.499
	5.268
	0.000
	0.000
	0.000
	0.000
	0.000

---

## 1.5 Example 5.

The data used in this example are given below.

---

```
close1 close3 close4 close5 close6 gender
851 851 851 851 851 851
37.9542 37.2785 37.0463 36.5696 36.1363 0.4900
6.3944 3.2716 4.1435 3.7058 4.1286 -0.0940
3.2716 7.5282 6.0804 5.1597 5.7608 -0.0390
4.1435 6.0804 10.7290 6.5672 7.2365 -0.1521
3.7058 5.1597 6.5672 10.2920 7.6463 -0.1104
4.1286 5.7608 7.2365 7.6463 12.9085 -0.1469
-0.0940 -0.0390 -0.1521 -0.1104 -0.1469 0.2502
```

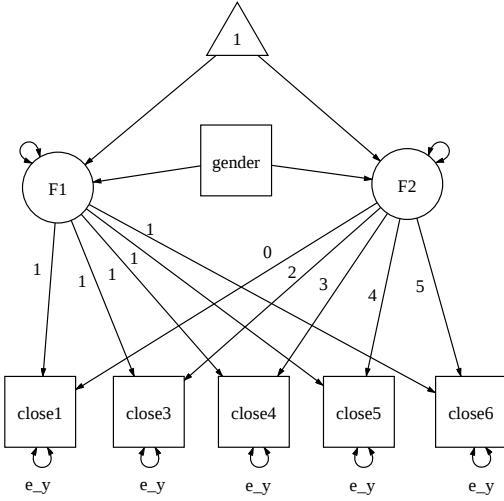
---

The output for the linear model with a covariate and with equal error variance in Figure 3a is given below.

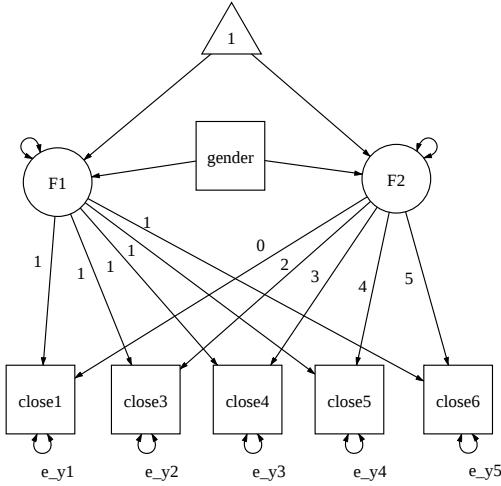
---

```
Test Statistics and Fit Indices
Fit for both mean and covariance
  Chi-square statistic    81.653
  Degrees of freedom      17
  p-value                  0
  Chi-square statistic (baseline model) 2272.4
  Degrees of freedom (baseline model)   23
  CFI                      0.971
  NFI                      0.964
  NNFI                     0.961
  RMSEA                    0.067
Fit for covariance only
  Chi-square statistic    76.586
  Degrees of freedom      14
  p-value                  0
  Chi-square statistic (baseline model) 1971.2
  Degrees of freedom (baseline model)   19
  CFI                      0.968
  NFI                      0.961
  NNFI                     0.956
  RMSEA                    0.072
Fit for mean only
  Chi-square statistic    5.073
```

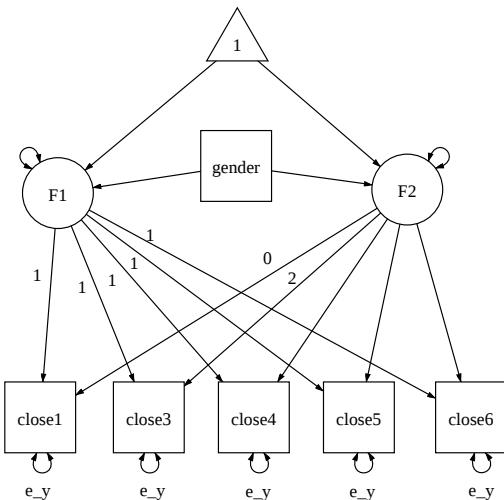
---



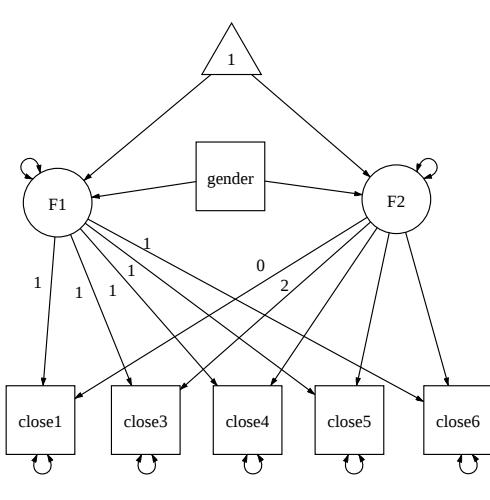
(a) Linear model with equal error variances



(b) Linear model with unequal error variances



(c) Nonlinear model with equal error variances



(d) Nonlinear model with unequal error variances

Figure 3: Path diagrams for growth curve models used in Example 5

Degrees of freedom	3
p-value	0.167
Chi-square statistic (baseline model)	287.88
Degrees of freedom (baseline model)	4
CFI	0.993
NFI	0.982
NNFI	0.99
RMSSEA	0.028
Parameter Estimates	
	Estimate Std. err Z-value P(> z )
Latent variables:	
F1 =~	
close1	1.000
close3	1.000
close4	1.000
close5	1.000
close6	1.000
F2 =~	
close1	0.000
close3	2.000
close4	3.000
close5	4.000
close6	5.000
Regressions:	
F1 ~	
gender	-0.289 0.163 -1.770 0.077
F2 ~	
gender	-0.052 0.043 -1.216 0.224
Covariances:	
F1 ~~	
F2	0.249 0.056 4.420 0.000
Intercepts:	
F2	-0.333 0.030 -11.169 0.000
F1	38.141 0.114 334.196 0.000
close1	0.000
close3	0.000
close4	0.000
close5	0.000
close6	0.000
Variances:	
close1 (e_y)	3.696 0.103 35.728 0.000
close3 (e_y)	3.696 0.103 35.728 0.000
close4 (e_y)	3.696 0.103 35.728 0.000
close5 (e_y)	3.696 0.103 35.728 0.000
close6 (e_y)	3.696 0.103 35.728 0.000
F1	2.956 0.284 10.399 0.000
F2	0.136 0.020 6.804 0.000

The output for the linear model with a covariate and with unequal error variance in Figure 3b is given below.

Test Statistics and Fit Indices  
 Fit for both mean and covariance  
     Chi-square statistic    58.463  
     Degrees of freedom      13  
     p-value                0  
     Chi-square statistic (baseline model) 2148.6  
     Degrees of freedom (baseline model) 19  
     CFI                   0.979  
     NFI                   0.973  
     NNFI                  0.969  
     RMSEA                0.064  
 Fit for covariance only  
     Chi-square statistic    53.862  
     Degrees of freedom      10  
     p-value                0  
     Chi-square statistic (baseline model) 1971.2  
     Degrees of freedom (baseline model) 15  
     CFI                   0.978  
     NFI                   0.973  
     NNFI                  0.966  
     RMSEA                0.072  
 Fit for mean only  
     Chi-square statistic    4.606  
     Degrees of freedom      3  
     p-value                0.203  
     Chi-square statistic (baseline model) 285  
     Degrees of freedom (baseline model) 4  
     CFI                   0.994  
     NFI                   0.984  
     NNFI                  0.992  
     RMSEA                0.025  
 Parameter Estimates  

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
F1 =~				
close1	1.000			
close3	1.000			
close4	1.000			
close5	1.000			
close6	1.000			
F2 =~				
close1	0.000			
close3	2.000			
close4	3.000			
close5	4.000			
close6	5.000			
Regressions:				
F1 ~				
gender	-0.274	0.162	-1.685	0.092
F2 ~				
gender	-0.052	0.043	-1.224	0.221

 Covariances:

F1	~~			
F2		0.242	0.065	3.713
<b>Intercepts:</b>				
F2		-0.330	0.030	-11.108
F1		38.128	0.114	335.599
close1		0.000		
close3		0.000		
close4		0.000		
close5		0.000		
close6		0.000		
<b>Variances:</b>				
close1 (e_y1)		3.522	0.275	12.811
close3 (e_y2)		2.986	0.182	16.385
close4 (e_y3)		4.000	0.236	16.945
close5 (e_y4)		3.660	0.238	15.411
close6 (e_y5)		4.504	0.318	14.145
F1		3.051	0.301	10.148
F2		0.127	0.023	5.607

The output for the nonlinear model with a covariate and with equal error variance in Figure 3c is given below.

---

```

Test Statistics and Fit Indices
Fit for both mean and covariance
  Chi-square statistic    78.959
  Degrees of freedom      14
  p-value                 0
  Chi-square statistic (baseline model) 2272.4
  Degrees of freedom (baseline model)   23
  CFI                     0.971
  NFI                     0.965
  NNFI                    0.953
  RMSEA                   0.074

Fit for covariance only
  Chi-square statistic    75.991
  Degrees of freedom      11
  p-value                 0
  Chi-square statistic (baseline model) 1971.2
  Degrees of freedom (baseline model)   19
  CFI                     0.967
  NFI                     0.961
  NNFI                    0.942
  RMSEA                   0.083

Fit for mean only
  Chi-square statistic    2.972
  Degrees of freedom      3
  p-value                 0.396
  Chi-square statistic (baseline model) 285.6
  Degrees of freedom (baseline model)   4
  CFI                     1
  NFI                     0.99
  NNFI                    1
  RMSEA                   0

```

Parameter Estimates				
	Estimate	Std. err	Z-value	P(> z )
<b>Latent variables:</b>				
F1 = ~				
close1	1.000			
close3	1.000			
close4	1.000			
close5	1.000			
close6	1.000			
F2 = ~				
close1	0.000			
close3	2.000			
close4	3.043	0.285	10.677	0.000
close5	4.027	0.374	10.769	0.000
close6	5.413	0.511	10.601	0.000
<b>Regressions:</b>				
F1 ~				
gender	-0.289	0.163	-1.781	0.075
F2 ~				
gender	-0.050	0.040	-1.238	0.216
<b>Covariances:</b>				
F1 ~~				
F2	0.235	0.056	4.225	0.000
<b>Intercepts:</b>				
F2	-0.312	0.043	-7.274	0.000
F1	38.114	0.117	325.411	0.000
close1	0.000			
close3	0.000			
close4	0.000			
close5	0.000			
close6	0.000			
<b>Variances:</b>				
close1 (e_y)	3.683	0.104	35.490	0.000
close3 (e_y)	3.683	0.104	35.490	0.000
close4 (e_y)	3.683	0.104	35.490	0.000
close5 (e_y)	3.683	0.104	35.490	0.000
close6 (e_y)	3.683	0.104	35.490	0.000
F1	3.035	0.295	10.302	0.000
F2	0.122	0.030	4.056	0.000

The output for the nonlinear model with a covariate and with equal error variance in Figure 3d is given below.

Test Statistics and Fit Indices	
Fit for both mean and covariance	
Chi-square statistic	57.928
Degrees of freedom	10
p-value	0
Chi-square statistic (baseline model)	2148.6
Degrees of freedom (baseline model)	19
CFI	0.977

NFI	0.973			
NNFI	0.957			
RMSEA	0.075			
Fit for covariance only				
Chi-square statistic	54.587			
Degrees of freedom	7			
p-value	0			
Chi-square statistic (baseline model)	1971.2			
Degrees of freedom (baseline model)	15			
CFI	0.976			
NFI	0.972			
NNFI	0.948			
RMSEA	0.089			
Fit for mean only				
Chi-square statistic	3.345			
Degrees of freedom	3			
p-value	0.341			
Chi-square statistic (baseline model)	284.99			
Degrees of freedom (baseline model)	4			
CFI	0.999			
NFI	0.988			
NNFI	0.998			
RMSEA	0.012			
Parameter Estimates				
	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
F1 = ~				
close1	1.000			
close3	1.000			
close4	1.000			
close5	1.000			
close6	1.000			
F2 = ~				
close1	0.000			
close3	2.000			
close4	2.988	0.266	11.245	0.000
close5	3.961	0.348	11.397	0.000
close6	5.138	0.464	11.075	0.000
Regressions:				
F1 ~				
gender	-0.274	0.162	-1.687	0.092
F2 ~				
gender	-0.052	0.042	-1.233	0.218
Covariances:				
F1 ~~				
F2	0.239	0.066	3.606	0.000
Intercepts:				
F2	-0.326	0.043	-7.541	0.000
F1	38.120	0.117	325.996	0.000
close1	0.000			
close3	0.000			

close4	0.000			
close5	0.000			
close6	0.000			
<b>Variances:</b>				
close1	3.523	0.275	12.817	0.000
close3	2.987	0.183	16.368	0.000
close4	4.009	0.239	16.763	0.000
close5	3.690	0.244	15.130	0.000
close6	4.431	0.340	13.033	0.000
F1	3.072	0.309	9.953	0.000
F2	0.124	0.032	3.863	0.000

---