Supplementary Materials for Yuan, Zhang, and Deng

October 16, 2017

1 Using WebSEM for Data Analysis

The fit indices for mean structures with growth curve models developed in this article are currently available in WebSEM, which is free online software that allows researchers to conduct SEM analysis through path diagrams (Zhang & Yuan, 2012-2017).[1] In this section, we illustrate how to obtain the fit indices in WebSEM through the linear model with equal error variances in Example 4 used in the paper. A video with instructions on how to conduct the data analysis is also available on YouTube (`https://youtu.be/1ydG3IxBIsM`).

To use WebSEM, one needs to provide a data file and draw a path diagram. The data file should be a free format text file with the extension name txt. The data file can be generated using any text editor or the default editor within WebSEM. Both raw data and summary data can be used. For missing data and non-normal data analysis, raw data are required. If raw data are used, the first line of the data file should be variable names and the rest are data entries. If summary data are used, the first row includes the variable names. The second row provides the value of the sample size that is repeated for each column of data. The third row includes information on the sample means. The rest is the full sample covariance matrix. For example, the summary data in the data file for Example 4 from Preacher et al. (2008) are given below. Clearly, there are six variables named close1, close2, close3, close4, close5 and gender, respectively, and the sample size is 851. Note that the value 851 repeats itself 6 times.

close1	close2	close3	close4	close5	gender
851	851	851	851	851	851
37.9542	37.2785	37.0463	36.5696	36.1363	0.4900
6.3944	3.2716	4.1435	3.7058	4.1286	-0.0940
3.2716	7.5282	6.0804	5.1597	5.7608	-0.0390
4.1435	6.0804	10.7290	6.5672	7.2365	-0.1521
3.7058	5.1597	6.5672	10.2920	7.6463	-0.1104
4.1286	5.7608	7.2365	7.6463	12.9085	-0.1469
-0.0940	-0.0390	-0.1521	-0.1104	-0.1469	0.2502

WebSEM provides an intuitive and interactive interface to draw path diagram for a model for data analysis. Information on how to use the interface can be found in Zhang and Yuan (2012-2017). The path diagram for the linear growth curve model drawn by WebSEM is given in Figure 2. The video on YouTube showed how to draw a path diagram interactively. Because the use of summary data instead of raw data, we put "`data=cov`" in the control field of WebSEM. Furthermore, to request the fit indices for the mean structure, type "`mean.fit`" in the control field; otherwise, only fit indices for the combined mean and covariance structure will be produced.

The output of WebSEM by running the analysis is shown below. Test Statistics and Fit Indices for both mean and covariance, covariance only, and mean only are given at the beginning of the output. For example, for evaluating both mean and covariance together, the chi-square statistic is 81.653 with 17 degrees of freedom. The corresponding p-value is essentially 0. The CFI is 0.971, NFI is 0.964, NNFI is 0.961 and RMSEA is 0.067. For mean only, the chi-square statistics is 5.073 (p-value = 0.167) and RMSEA is 0.028.

---

[1] Zhang, Z. & Yuan, K.-H. (2012-2017). WebSEM: Structural equation modeling online [Manual available at http://psychstat.org/websem]. Retrievable from https://websem.psychstat.org.
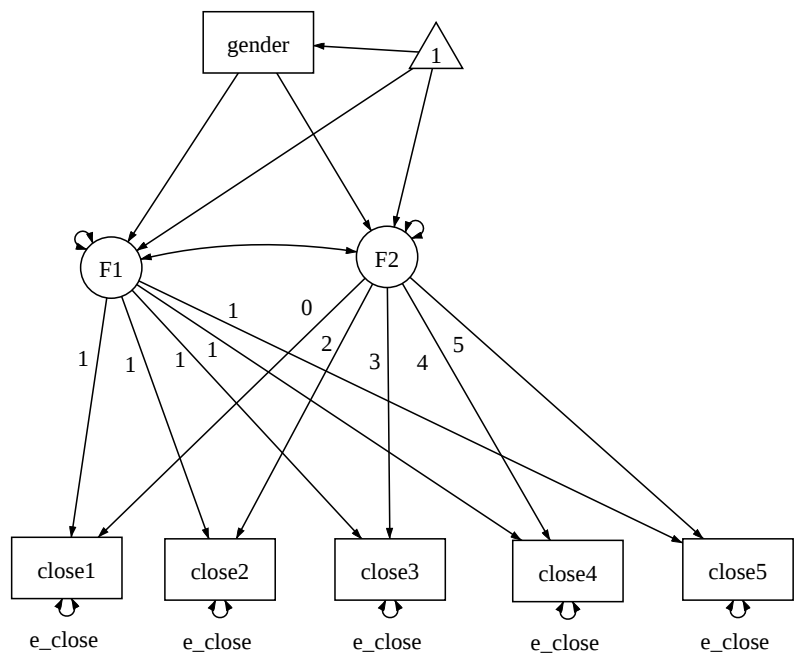
Figure 1: Path diagram for the linear growth curve model with equal error variances. The path diagram was drawn by WebSEM.

```
Model information
Observed variables: gender close1 close2 close3 close4 close5 .
Latent variables: F1 F2 .
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
          Chi-square statistic   81.647
          Degrees of freedom     17
          p-value          0
          Chi-square statistic (baseline model)  2272.2
          Degrees of freedom (baseline model)    23
          CFI             0.971
          NFI             0.964
          NNFI            0.961
          RMSEA           0.067
Fit for covariance only
          Chi-square statistic   76.586
          Degrees of freedom     14
          p-value          0
          Chi-square statistic (baseline model)  2104.7
          Degrees of freedom (baseline model)    19
          CFI             0.97
          NFI             0.964
          NNFI            0.959
          RMSEA           0.072
Fit for mean only
          Chi-square statistic   5.067
          Degrees of freedom     3
          p-value          0.167
          Chi-square statistic (baseline model)  287.54
          Degrees of freedom (baseline model)    4
          CFI             0.993
          NFI             0.982
          NNFI            0.99
          RMSEA           0.028
Parameter Estimates
                    Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1             1.000
    close2             1.000
    close3             1.000
    close4             1.000
    close5             1.000
  F2 =~
    close1             0.000
    close2             2.000
    close3             3.000
    close4             4.000
    close5             5.000

Regressions:
```

```
    F1 ~
       gender               -0.289      0.163     -1.769      0.077
    F2 ~
       gender               -0.052      0.043     -1.215      0.224

 Covariances:
   F1 ~~
       F2                    0.250      0.057      4.417      0.000

 Intercepts:
       F1                   38.141      0.114    333.899      0.000
       F2                   -0.333      0.030    -11.159      0.000
       gender                0.490      0.017     28.560      0.000
       close1                0.000
       close2                0.000
       close3                0.000
       close4                0.000
       close5                0.000

 Variances:
       F1                    2.960      0.285     10.393      0.000
       F2                    0.136      0.020      6.800      0.000
       close1 (e_cl)         3.700      0.104     35.707      0.000
       close2 (e_cl)         3.700      0.104     35.707      0.000
       close3 (e_cl)         3.700      0.104     35.707      0.000
       close4 (e_cl)         3.700      0.104     35.707      0.000
       close5 (e_cl)         3.700      0.104     35.707      0.000
       gender                0.250      0.012     20.616      0.000


 =======================================
 WebSEM ended at 16:35:01 on Oct 16, 2017
```

We now include the input and output of the examples used in our paper.

1.1 Example 1

The summary data used in this example are given below. The model in Figure 2 is used.

```
 y1      y2      y3      y4      y5      y6
200     200     200     200     200     200
1.0     1.5     2.0     2.5     3.0     3.5
2.0     2.0     2.0     2.5     3.0     3.5
2.0     4.0     4.5     6.0     7.5     9.0
2.0     4.5     8.0    10.0    12.0    14.5
2.5     6.0    10.0    14.0    16.5    20.0
3.0     7.5    12.0    16.5    22.0    26.0
3.5     9.0    14.5    20.0    26.0    32.0
```
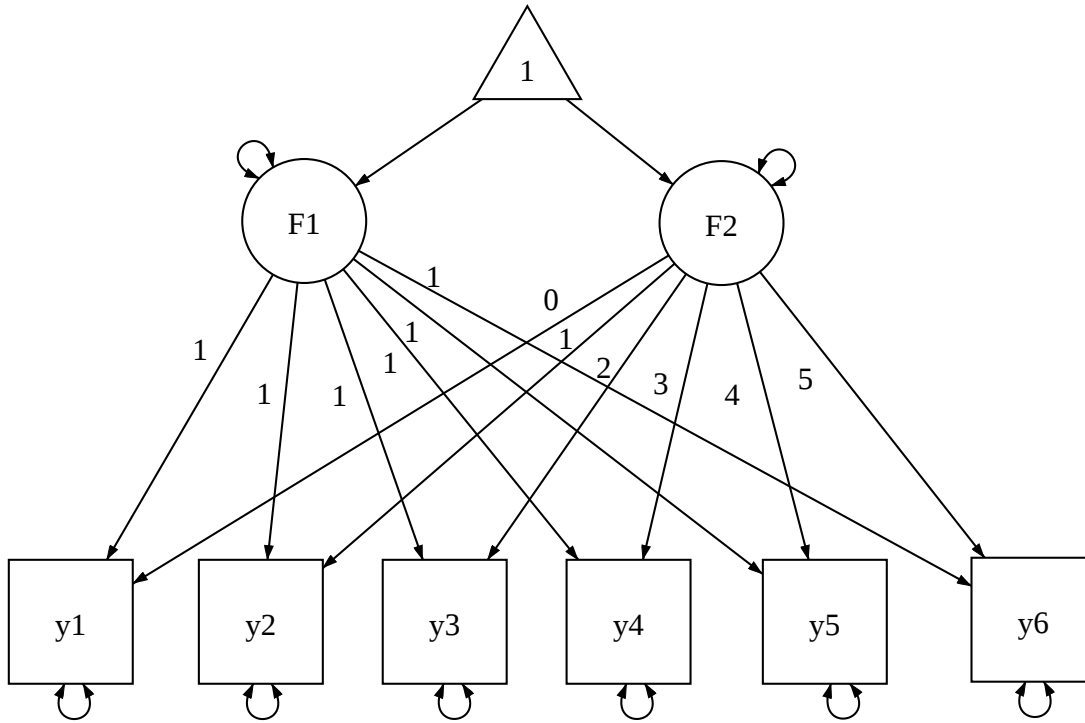
Figure 2: Path diagrams for models used in Examples 1-4

The output of WebSEM by running the analysis is shown below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    82.686
        Degrees of freedom      16
        p-value         0
        Chi-square statistic (baseline model)  2010.2
        Degrees of freedom (baseline model)    20
        CFI             0.966
        NFI             0.959
        NNFI            0.958
        RMSEA           0.144
Fit for covariance only
        Chi-square statistic    82.686
        Degrees of freedom      12
        p-value         0
        Chi-square statistic (baseline model)  1923.8
        Degrees of freedom (baseline model)    15
        CFI             0.963
        NFI             0.957
        NNFI            0.954
        RMSEA           0.172
Fit for mean only
        Chi-square statistic    0
        Degrees of freedom      4
        p-value         1
        Chi-square statistic (baseline model)  46.273
```

```
          Degrees of freedom (baseline model)     5
          CFI                 1
          NFI                 1
          NNFI              1.121
          RMSEA               0

Parameter Estimates
                     Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    y1                  1.000
    y2                  1.000
    y3                  1.000
    y4                  1.000
    y5                  1.000
  F2 =~
    y1                  0.000
    y2                  1.000
    y3                  2.000
    y4                  3.000
    y5                  4.000
  F1 =~
    y6                  1.000
  F2 =~
    y6                  5.000

Covariances:
  F1 ~~
    F2      (r_F1)      0.378     0.100     3.779     0.000

Intercepts:
    F2                  0.500     0.073     6.819     0.000
    F1                  1.000     0.094    10.667     0.000
    y1                  0.000
    y2                  0.000
    y3                  0.000
    y4                  0.000
    y5                  0.000
    y6                  0.000

Variances:
    y1                  0.532     0.103     5.186     0.000
    y2                  0.642     0.085     7.590     0.000
    y3                  1.040     0.119     8.740     0.000
    y4                  1.087     0.125     8.715     0.000
    y5                  0.595     0.090     6.627     0.000
    F1                  1.444     0.181     7.970     0.000
    F2                  1.044     0.108     9.710     0.000
    y6                  0.483     0.127     3.807     0.000
```

1.2   Example 2

The summary data used in this example are given below. The same model in Figure 2 is used.

```
y1      y2      y3      y4      y5      y6
200.0   200.0   200.0   200.0   200.0   200.0
1.0     1.5     2.0     2.5     3.0     3.5
2.0     2.2     2.0     2.5     3.0     3.5
2.2     4.0     4.5     6.0     7.5     9.0
2.0     4.5     8.0     10.2    12.0    14.5
2.5     6.0     10.2    14.0    16.5    20.0
3.0     7.5     12.0    16.5    22.0    26.2
3.5     9.0     14.5    20.0    26.2    32.0
```

The output of the analysis is given below.

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    188.53
        Degrees of freedom      16
        p-value         0
        Chi-square statistic (baseline model)   2255.5
        Degrees of freedom (baseline model)     20
        CFI             0.923
        NFI             0.916
        NNFI            0.904
        RMSEA           0.232
 Fit for covariance only
        Chi-square statistic    188.53
        Degrees of freedom      12
        p-value         0
        Chi-square statistic (baseline model)   2168.6
        Degrees of freedom (baseline model)     15
        CFI             0.918
        NFI             0.913
        NNFI            0.898
        RMSEA           0.271
 Fit for mean only
        Chi-square statistic    0
        Degrees of freedom      4
        p-value         1
        Chi-square statistic (baseline model)   46.193
        Degrees of freedom (baseline model)     5
        CFI             1
        NFI             1
        NNFI            1.122
        RMSEA           0
Parameter Estimates
                   Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    y1                1.000
    y2                1.000
    y3                1.000
    y4                1.000
    y5                1.000
  F2 =~
    y1                0.000
    y2                1.000
```

```
      y3                    2.000
      y4                    3.000
      y5                    4.000
   F1 =~
      y6                    1.000
   F2 =~
      y6                    5.000

 Covariances:
   F1 ~~
      F2      (r_F1)    0.326    0.103    3.160    0.002

 Intercepts:
      F2                0.500    0.074    6.797    0.000
      F1                1.000    0.097   10.321    0.000
      y1                0.000
      y2                0.000
      y3                0.000
      y4                0.000
      y5                0.000
      y6                0.000

 Variances:
      y1                0.288    0.073    3.957    0.000
      y2                0.438    0.058    7.525    0.000
      y3                1.188    0.130    9.115    0.000
      y4                1.226    0.134    9.180    0.000
      y5                0.405    0.064    6.311    0.000
      F1                1.687    0.192    8.793    0.000
      F2                1.065    0.108    9.838    0.000
      y6                0.255    0.093    2.751    0.006
```

1.3   Example 3

The summary data for Example 3 are given below.

```
 y1      y2      y3      y4      y5      y6
 200.0   200.0   200.0   200.0   200.0   200.0
 1.2     1.5     2.0     2.5     3.0     3.2
 2.0     1.5     2.0     2.5     3.0     3.5
 1.5     4.0     4.5     6.0     7.5     9.0
 2.0     4.5     8.0     9.5    12.0    14.5
 2.5     6.0     9.5    14.0    16.5    20.0
 3.0     7.5    12.0    16.5    22.0    25.5
 3.5     9.0    14.5    20.0    25.5    32.0
```

The linear growth curve model in Figure 2 is first fitted to the data and the output is given below.

```
 Test Statistics and Fit Indices
 Fit for both mean and covariance
         Chi-square statistic   33.093
         Degrees of freedom      16
         p-value          7e-03
         Chi-square statistic (baseline model)  1791.9
         Degrees of freedom (baseline model)     20
```

8

```
         CFI              0.99
         NFI              0.982
         NNFI             0.988
         RMSEA            0.073
Fit for covariance only
         Chi-square statistic   13.121
         Degrees of freedom      12
         p-value          0.36
         Chi-square statistic (baseline model)  1650.6
         Degrees of freedom (baseline model)    15
         CFI              0.999
         NFI              0.992
         NNFI             0.999
         RMSEA            0.022
Fit for mean only
         Chi-square statistic   20.073
         Degrees of freedom      4
         p-value          0
         Chi-square statistic (baseline model)  99.134
         Degrees of freedom (baseline model)    5
         CFI              0.829
         NFI              0.798
         NNFI             0.787
         RMSEA            0.142
Parameter Estimates
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    y1               1.000
    y2               1.000
    y3               1.000
    y4               1.000
    y5               1.000
  F2 =~
    y1               0.000
    y2               1.000
    y3               2.000
    y4               3.000
    y5               4.000
  F1 =~
    y6               1.000
  F2 =~
    y6               5.000


Covariances:
  F1 ~~
    F2               0.499    0.094    5.327    0.000


Intercepts:
    F2               0.429    0.073    5.902    0.000
    F1               1.162    0.087   13.304    0.000
    y1               0.000
    y2               0.000
    y3               0.000
```

```
        y4                      0.000
        y5                      0.000
        y6                      0.000

Variances:
        y1                      0.999     0.146     6.819     0.000
        y2                      1.007     0.122     8.223     0.000
        y3                      0.994     0.116     8.599     0.000
        y4                      0.995     0.122     8.127     0.000
        y5                      1.018     0.151     6.761     0.000
        F1                      0.991     0.158     6.266     0.000
        F2                      0.994     0.105     9.440     0.000
        y6                      1.024     0.204     5.015     0.000
```

The nonlinear growth curve model in Figure 3 below is then fitted to the data and the output is given below.

Figure 3: Qudratic growth curve model with unequal variances and a covariate

```
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    4.849
        Degrees of freedom      12
        p-value         0.963
```

```
              Chi-square statistic (baseline model)  1791.9
              Degrees of freedom (baseline model)    20
              CFI              1
              NFI              0.997
              NNFI             1.007
              RMSEA            0
Fit for covariance only
              Chi-square statistic   0.814
              Degrees of freedom     9
              p-value          1
              Chi-square statistic (baseline model)  1650.6
              Degrees of freedom (baseline model)    15
              CFI              1
              NFI              1
              NNFI             1.008
              RMSEA            0
Fit for mean only
              Chi-square statistic   4.056
              Degrees of freedom     3
              p-value          0.256
              Chi-square statistic (baseline model)  91.106
              Degrees of freedom (baseline model)    5
              CFI              0.988
              NFI              0.955
              NNFI             0.98
              RMSEA            0.042
Parameter Estimates
                      Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    y1                  1.000
    y2                  1.000
    y3                  1.000
    y4                  1.000
    y5                  1.000
  F2 =~
    y1                  0.000
    y2                  1.000
    y3                  2.079    0.178    11.705    0.000
    y4                  3.159    0.276    11.435    0.000
    y5                  4.239    0.380    11.168    0.000
  F1 =~
    y6                  1.000
  F2 =~
    y6                  5.193    0.472    10.993    0.000

Covariances:
  F1 ~~
    F2                  0.490    0.092    5.313     0.000

Intercepts:
    F2                  0.410    0.080    5.143     0.000
    F1                  1.165    0.090    12.987    0.000
    y1                  0.000
```

```
         y2                      0.000
         y3                      0.000
         y4                      0.000
         y5                      0.000
         y6                      0.000

Variances:
         y1                      1.003    0.147    6.827    0.000
         y2                      1.002    0.122    8.180    0.000
         y3                      0.994    0.116    8.600    0.000
         y4                      0.990    0.122    8.085    0.000
         y5                      0.989    0.152    6.490    0.000
         F1                      1.005    0.165    6.081    0.000
         F2                      0.899    0.200    4.493    0.000
         y6                      1.064    0.208    5.115    0.000
```

1.4   Example 4

The summary data for Example 4 are given below.

```
close1    close2    close3    close4    close5    gender
851       851       851       851       851       851
37.9542   37.2785   37.0463   36.5696   36.1363   0.4900
6.3944    3.2716    4.1435    3.7058    4.1286    -0.0940
3.2716    7.5282    6.0804    5.1597    5.7608    -0.0390
4.1435    6.0804    10.7290   6.5672    7.2365    -0.1521
3.7058    5.1597    6.5672    10.2920   7.6463    -0.1104
4.1286    5.7608    7.2365    7.6463    12.9085   -0.1469
-0.0940   -0.0390   -0.1521   -0.1104   -0.1469   0.2502
```

4.1. The linear growth curve model with homogeneous variance in Figure 4 is first fitted to the data and the output is given below.

```
Model information
Observed variables: gender close1 close2 close3 close4 close5 .
Latent variables: F1 F2 .
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
         Chi-square statistic    81.647
         Degrees of freedom      17
         p-value         0
         Chi-square statistic (baseline model)   2272.2
         Degrees of freedom (baseline model)     23
         CFI             0.971
         NFI             0.964
         NNFI            0.961
         RMSEA           0.067
Fit for covariance only
         Chi-square statistic    76.586
         Degrees of freedom      14
         p-value         0
         Chi-square statistic (baseline model)   2104.7
         Degrees of freedom (baseline model)     19
```
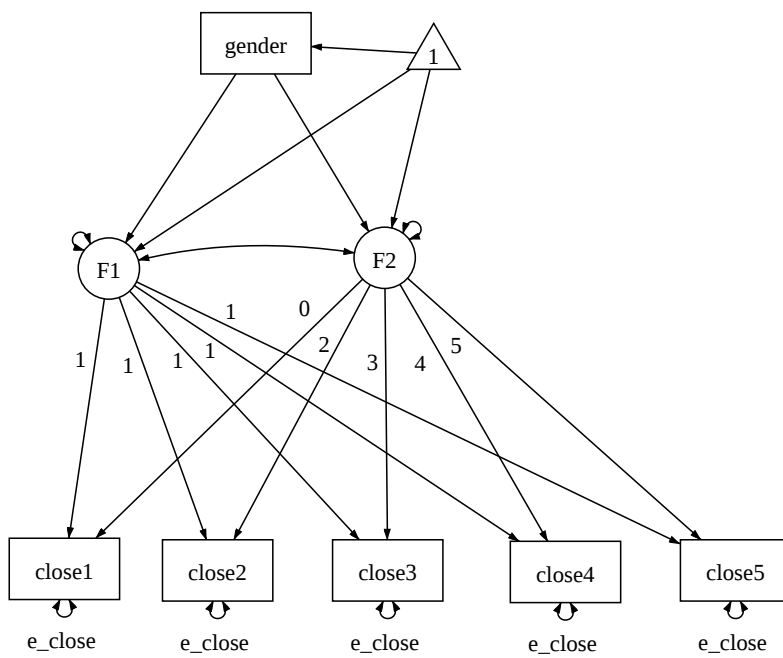
Figure 4: Linear growth curve model with equal variances and a covariate

```
        CFI                 0.97
        NFI                 0.964
        NNFI                0.959
        RMSEA               0.072
Fit for mean only
        Chi-square statistic   5.067
        Degrees of freedom     3
        p-value             0.167
        Chi-square statistic (baseline model)   287.54
        Degrees of freedom (baseline model)      4
        CFI                 0.993
        NFI                 0.982
        NNFI                0.99
        RMSEA               0.028
Parameter Estimates
                   Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    close1              1.000
    close2              1.000
    close3              1.000
    close4              1.000
    close5              1.000
  F2 =~
    close1              0.000
    close2              2.000
    close3              3.000
    close4              4.000
    close5              5.000

Regressions:
  F1 ~
    gender             -0.289    0.163    -1.769    0.077
  F2 ~
    gender             -0.052    0.043    -1.215    0.224

Covariances:
  F1 ~~
    F2                  0.250    0.057     4.417    0.000

Intercepts:
    F1                 38.141    0.114   333.899    0.000
    F2                 -0.333    0.030   -11.159    0.000
    gender              0.490    0.017    28.560    0.000
    close1              0.000
    close2              0.000
    close3              0.000
    close4              0.000
    close5              0.000

Variances:
    F1                  2.960    0.285    10.393    0.000
    F2                  0.136    0.020     6.800    0.000
    close1 (e_cl)       3.700    0.104    35.707    0.000
```
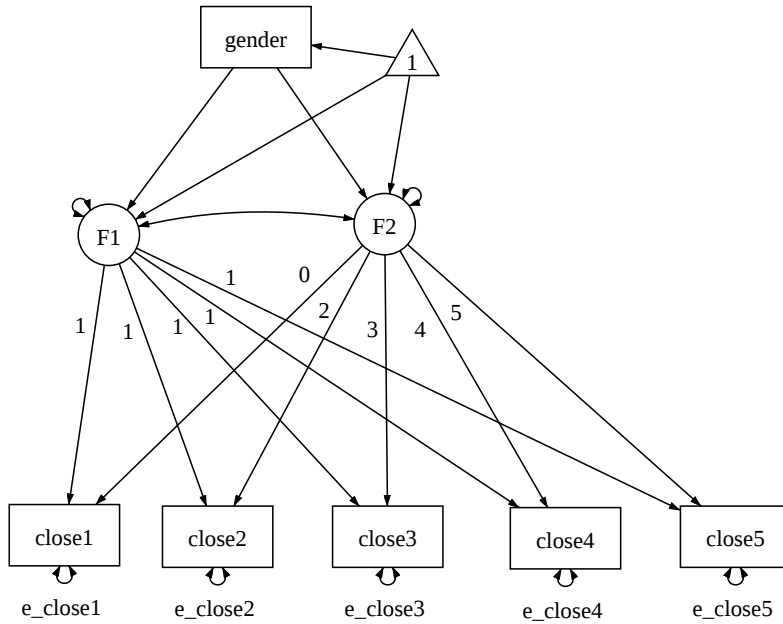
Figure 5: Linear growth curve model with unequal variances and a covariate

```
        close2 (e_cl)       3.700      0.104      35.707      0.000
        close3 (e_cl)       3.700      0.104      35.707      0.000
        close4 (e_cl)       3.700      0.104      35.707      0.000
        close5 (e_cl)       3.700      0.104      35.707      0.000
        gender              0.250      0.012      20.616      0.000


        =====================================
        WebSEM ended at 16:35:01 on Oct 16, 2017
```

4.2. Linear model with unequal variances.

```
Model information
Observed variables: gender close1 close2 close3 close4 close5 .
Latent variables: F1 F2 .
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    58.457
        Degrees of freedom      13
        p-value         0
        Chi-square statistic (baseline model)   2148.4
        Degrees of freedom (baseline model)     19
        CFI             0.979
```

```
                  NFI             0.973
                  NNFI            0.969
                  RMSEA           0.064
Fit for covariance only
          Chi-square statistic   53.862
          Degrees of freedom     10
          p-value         0
          Chi-square statistic (baseline model)   1971.2
          Degrees of freedom (baseline model)     15
          CFI             0.978
          NFI             0.973
          NNFI            0.966
          RMSEA           0.072
Fit for mean only
          Chi-square statistic   4.601
          Degrees of freedom     3
          p-value         0.203
          Chi-square statistic (baseline model)   284.67
          Degrees of freedom (baseline model)     4
          CFI             0.994
          NFI             0.984
          NNFI            0.992
          RMSEA           0.025
Parameter Estimates
                    Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1            1.000
    close2            1.000
    close3            1.000
    close4            1.000
    close5            1.000
  F2 =~
    close1            0.000
    close2            2.000
    close3            3.000
    close4            4.000
    close5            5.000

Regressions:
  F1 ~
    gender           -0.274    0.162    -1.686    0.092
  F2 ~
    gender           -0.052    0.042    -1.224    0.221

Covariances:
  F1 ~~
    F2                0.242    0.068     3.572    0.000

Intercepts:
    F1               38.128    0.114   335.436    0.000
    F2               -0.330    0.030   -11.102    0.000
    gender            0.490    0.017    28.560    0.000
    close1            0.000
```

Figure 6: Quadratic growth curve model with equal variances and a covariate

```
    close2                  0.000
    close3                  0.000
    close4                  0.000
    close5                  0.000

Variances:
    F1                      3.055     0.314     9.742     0.000
    F2                      0.127     0.023     5.544     0.000
    close1 (e_c1)           3.527     0.289    12.219     0.000
    close2 (e_c2)           2.989     0.182    16.470     0.000
    close3 (e_c3)           4.004     0.235    17.056     0.000
    close4 (e_c4)           3.665     0.237    15.471     0.000
    close5 (e_c5)           4.509     0.318    14.194     0.000
    gender                  0.250     0.012    20.616     0.000


=======================================
WebSEM ended at 16:36:09 on Oct 16, 2017
```

4.3 The nonlinear growth curve model with equal variance in Figure 6 is then fitted to the data and the output is given below.

```
Model information
Observed variables: gender close1 close2 close3 close4 close5 .
Latent variables: F1 F2 F3 .
```

```
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic   53.631
        Degrees of freedom      12
        p-value          0
        Chi-square statistic (baseline model)  2272.2
        Degrees of freedom (baseline model)    23
        CFI            0.981
        NFI            0.976
        NNFI           0.965
        RMSEA          0.064
Fit for covariance only
        Chi-square statistic   51.52
        Degrees of freedom      10
        p-value          0
        Chi-square statistic (baseline model)  2104.7
        Degrees of freedom (baseline model)    19
        CFI            0.98
        NFI            0.976
        NNFI           0.962
        RMSEA          0.07
Fit for mean only
        Chi-square statistic   2.114
        Degrees of freedom      2
        p-value          0.348
        Chi-square statistic (baseline model)  284.93
        Degrees of freedom (baseline model)    4
        CFI            1
        NFI            0.993
        NNFI           0.999
        RMSEA          8e-03
Parameter Estimates
                    Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    close1             1.000
    close2             1.000
    close3             1.000
    close4             1.000
    close5             1.000
  F2 =~
    close1             0.000
    close2             2.000
    close3             3.000
    close4             4.000
    close5             5.000
  F3 =~
    close1             0.000
    close2             4.000
    close3             9.000
    close4             16.000
    close5             25.000
```

```
Regressions:
  F1 ~
    gender              -0.341     0.171    -2.001     0.045
  F2 ~
    gender               0.036     0.120     0.296     0.767
  F3 ~
    gender              -0.018     0.023    -0.760     0.447

Covariances:
  F1 ~~
    F2                   0.305     0.175     1.745     0.081
  F2 ~~
    F3                  -0.101     0.032    -3.140     0.002
  F1 ~~
    F3                  -0.014     0.032    -0.452     0.652

Intercepts:
    F1                  38.110     0.119   318.989     0.000
    F2                  -0.281     0.084    -3.343     0.001
    gender               0.490     0.017    28.560     0.000
    F3                  -0.011     0.016    -0.643     0.520
    close1               0.000
    close2               0.000
    close3               0.000
    close4               0.000
    close5               0.000

Variances:
    F1                   2.880     0.321     8.970     0.000
    F2                   0.587     0.171     3.426     0.001
    close1 (e_cl)        3.412     0.117    29.155     0.000
    close2 (e_cl)        3.412     0.117    29.155     0.000
    close3 (e_cl)        3.412     0.117    29.155     0.000
    close4 (e_cl)        3.412     0.117    29.155     0.000
    close5 (e_cl)        3.412     0.117    29.155     0.000
    F3                   0.023     0.006     3.578     0.000
    gender               0.250     0.012    20.616     0.000


=======================================
WebSEM ended at 16:37:38 on Oct 16, 2017
```

4.4 Nonlinear model with unequal variances

```
Model information
Observed variables: gender close1 close2 close3 close4 close5 .
Latent variables: F1 F2 F3 .
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic   26.958
        Degrees of freedom      8
        p-value              1e-03
```
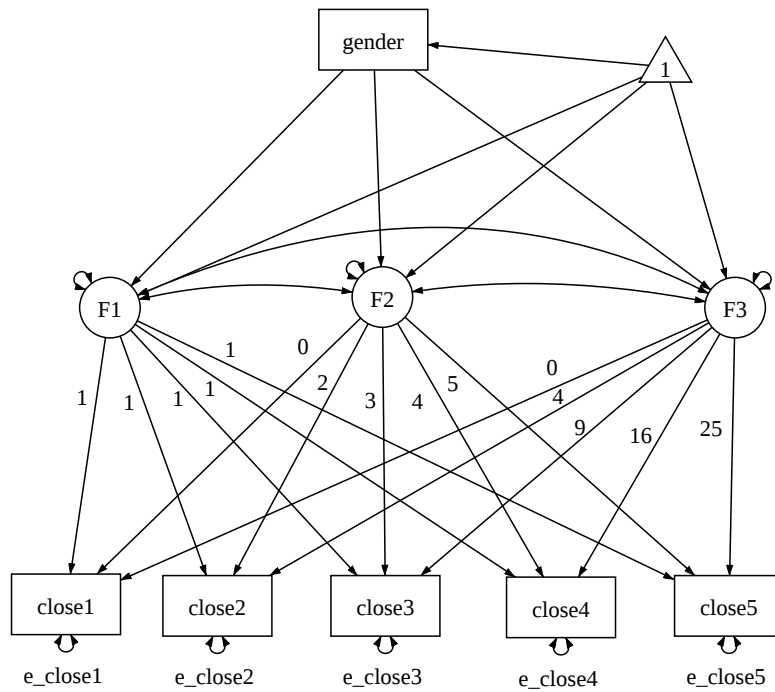
Figure 7: Quadratic growth curve model with unequal variances and a covariate

```
              Chi-square statistic (baseline model)   2148.4
              Degrees of freedom (baseline model)      19
              CFI             0.991
              NFI             0.987
              NNFI            0.979
              RMSEA           0.053
Fit for covariance only
              Chi-square statistic    24.669
              Degrees of freedom       6
              p-value          0
              Chi-square statistic (baseline model)   1971.2
              Degrees of freedom (baseline model)      15
              CFI             0.99
              NFI             0.987
              NNFI            0.976
              RMSEA           0.06
Fit for mean only
              Chi-square statistic    2.291
              Degrees of freedom       2
              p-value          0.318
              Chi-square statistic (baseline model)   282.64
              Degrees of freedom (baseline model)       4
              CFI             0.999
              NFI             0.992
              NNFI            0.998
              RMSEA           0.013
Parameter Estimates
                    Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    close1              1.000
    close2              1.000
    close3              1.000
    close4              1.000
    close5              1.000
  F2 =~
    close1              0.000
    close2              2.000
    close3              3.000
    close4              4.000
    close5              5.000
  F3 =~
    close1              0.000
    close2              4.000
    close3              9.000
    close4             16.000
    close5             25.000

Regressions:
  F1 ~
    gender             -0.319     0.170    -1.871     0.061
  F2 ~
    gender              0.053     0.120     0.443     0.658
  F3 ~
```

```
        gender              -0.023    0.023   -0.972    0.331

Covariances:
  F1 ~~
    F2                       1.162    0.472    2.463    0.014
  F2 ~~
    F3                      -0.052    0.049   -1.066    0.287
  F1 ~~
    F3                      -0.135    0.069   -1.946    0.052

Intercepts:
    F1                      38.092    0.119  319.590    0.000
    F2                      -0.295    0.084   -3.521    0.000
    gender                   0.490    0.017   28.560    0.000
    F3                      -0.007    0.016   -0.405    0.686
    close1                   0.000
    close2                   0.000
    close3                   0.000
    close4                   0.000
    close5                   0.000

Variances:
    F1                       1.546    0.764    2.023    0.043
    F2                       0.154    0.315    0.490    0.624
    close1 (e_c1)            4.839    0.773    6.259    0.000
    close2 (e_c2)            2.394    0.197   12.157    0.000
    close3 (e_c3)            3.723    0.232   16.023    0.000
    close4 (e_c4)            3.924    0.247   15.870    0.000
    close5 (e_c5)            3.243    0.451    7.195    0.000
    F3                       0.019    0.009    2.242    0.025
    gender                   0.250    0.012   20.616    0.000



=======================================
WebSEM ended at 16:38:34 on Oct 16, 2017
```

1.5 Missing data and non-normal data

For the analysis of missing data and non-normal data, the raw data are needed. The data used in this example are given below.

```
piat1997 piat1998 piat1999 piat2000 income1 fgrade mgrade asvab
NA 65 50 61 NA 10 15 8.334
48 44 50 NA NA NA 8 NA
60 37 42 52 NA NA 8 NA
61 85 69 71 17.626 12 12 47.134
NA 38 72 69 8 NA 14 58.541
59 79 61 57 NA 14 14 52.921
52 72 79 69 NA NA 95 27.471
73 79 82 73 2.076 12 11 NA
41 67 NA NA NA 12 12 2.39
64 69 72 NA NA 12 12 NA
73 74 82 NA NA 12 12 NA
69 78 72 70 42 12 16 39.585
```

```
51 45 50 70 1 12 12 18.371
63 72 69 74 NA NA 11 12.233
80 85 89 92 23 NA 11 53.768
78 81 96 97 NA 12 14 56.63
74 50 NA 62 NA 12 11 NA
80 65 85 86 5.8 16 13 NA
63 72 64 NA NA 11 14 26.195
68 86 73 90 6 NA 15 27.912
82 92 95 97 NA 15 16 77.604
61 58 79 72 34 12 12 27.664
76 55 69 82 0.201 12 10 NA
80 81 87 85 50 16 18 51.582
50 48 61 73 2 12 9 16.523
60 55 62 NA 26 16 13 NA
76 82 NA NA NA 14 18 70.421
64 NA 57 65 NA 12 NA NA
93 93 95 97 80 16 16 97.811
50 58 77 76 NA NA NA 9.674
49 54 72 65 37 12 14 26.572
58 62 81 81 14 16 16 NA
60 70 72 82 10 NA NA 12.534
67 90 84 72 25 NA 12 35.74
68 80 85 90 50 12 9 22.08
66 59 60 70 40 12 14 2.167
75 76 81 85 12.978 NA NA 14.279
90 89 97 95 NA 12 16 NA
73 76 81 79 NA 7 16 29.276
44 50 NA 55 NA NA NA 0.835
31 NA 28 NA NA NA 11 12.321
60 56 67 65 0 12 12 17.119
NA NA 87 74 NA 16 9 46.48
79 68 98 88 26 11 12 30.329
92 80 91 NA NA 3 6 34.815
35 62 70 74 NA 18 13 15.719
39 50 71 NA NA NA 9 14.495
61 88 63 83 NA 12 15 94.837
70 63 64 70 13 14 12 NA
69 53 65 66 13 NA 11 2.675
52 54 49 NA NA NA 12 NA
60 67 75 70 0 NA 12 5.648
24 48 42 52 0.346 NA NA 10.056
40 50 66 67 NA NA 12 47.984
NA 29 49 49 NA NA 12 6.732
55 63 79 69 NA 16 16 76.56
60 47 52 49 3.2 14 15 28.346
54 64 78 68 3 NA 14 NA
61 75 77 93 NA 12 12 NA
58 47 61 50 51.5 12 15 77.556
40 56 51 62 12.48 NA NA NA
85 74 100 92 25 11 12 NA
60 57 83 77 17 NA 9 NA
58 NA 49 55 20 8 14 11.463
60 88 89 89 0 13 13 78.616
75 73 74 82 30 11 10 NA
```

```
54 45 43 57 NA 11 10 NA
44 NA 46 73 NA 12 12 71.217
30 57 30 59 NA NA 8 25.968
70 NA 94 NA 25 14 14 NA
51 59 49 25 NA 9 10 42.665
84 89 95 93 19 16 16 81.228
58 67 NA 83 0.25 18 18 62.397
61 60 65 89 NA 12 12 24.844
56 51 61 60 NA 12 12 37.476
65 68 72 66 11 NA 12 14.151
66 73 NA 69 13 17 18 27.771
58 58 60 74 10 13 14 49.108
39 53 NA NA NA NA 12 19.208
59 76 50 NA 0.8 15 12 20.767
60 NA 60 NA NA 12 12 NA
79 74 74 66 19 NA 10 23.715
62 74 72 NA NA 13 13 13.75
NA 66 49 66 NA 16 16 NA
85 88 91 97 16 12 13 45.173
68 NA 68 70 14.4 NA 11 4.148
58 49 54 39 NA 12 NA 1.523
50 62 50 74 10 12 11 NA
78 93 NA 93 54.56 12 12 18.284
58 75 61 52 8 4 6 0.578
63 59 83 67 14 NA 12 NA
67 83 90 92 21 12 9 30.856
79 78 86 94 10 12 12 NA
49 58 66 56 23 12 12 NA
50 70 64 68 174.605 12 14 57.45
48 60 56 52 NA 12 14 78.004
97 92 72 71 17 NA NA 97.332
70 NA 78 74 NA 15 14 17.147
67 74 NA NA 3 NA 12 4.378
41 65 66 60 32 15 12 NA
95 96 95 99 21 12 12 3.638
NA 86 88 93 23 NA 15 80.289
NA 60 67 NA NA 12 14 54.581
66 71 72 NA 26 16 16 68.893
74 NA 88 89 28 16 12 10.889
49 64 61 55 NA NA 12 NA
55 42 27 41 NA 16 16 90.539
63 50 52 69 6 16 16 11.417
NA 59 75 65 NA NA NA 57.619
42 71 NA NA 42.5 14 16 30.579
76 83 80 86 0.013 11 11 19.265
67 64 63 92 48 12 11 60.68
69 70 73 76 15 12 12 17.255
56 NA 43 68 NA 12 14 20.272
71 55 72 66 NA 12 12 36.293
60 37 88 78 42 10 12 47.193
83 73 NA NA 6 12 14 68.082
69 66 61 73 NA 16 NA 89.749
71 85 74 70 21 9 14 18.671
62 65 NA 22 18.9 NA 12 NA
```

```
53 65 67 73 30.5 13 NA 36.962
NA 41 34 NA NA NA 12 23.558
NA 72 66 73 NA 16 12 18.95
51 63 60 67 16 12 12 32.051
56 68 75 NA 35 14 12 NA
52 62 70 60 20 NA 13 10.672
16 20 23 26 NA 10 13 71.841
60 54 62 77 NA NA NA 72.105
88 96 89 NA NA 16 16 NA
56 65 75 83 0 12 10 80.897
85 89 92 NA NA 10 12 0.297
66 70 66 69 NA NA 11 NA
72 73 74 60 30 16 16 92.602
68 82 79 77 46.6 NA 14 61.967
60 67 66 81 5 NA 11 3.172
43 62 NA 58 5 NA 11 9.832
55 47 34 65 NA NA 11 19.029
80 90 89 64 33 NA NA 59.867
67 87 99 87 0 12 13 68.917
79 88 73 92 0.03 12 12 30.257
81 78 82 87 23 NA 12 39.821
65 85 75 88 NA 12 12 8.281
28 2 54 61 NA 8 14 14.754
88 87 90 97 NA 12 12 1.778
75 76 83 87 21 16 18 NA
65 63 69 63 2 16 16 48.117
75 73 79 81 NA 12 12 75.756
70 72 85 89 8.5 6 11 43.971
58 46 53 70 19 NA 12 8.901
83 87 86 97 NA NA NA 43.183
66 69 68 89 14.556 12 12 28.106
65 59 66 73 1.5 12 12 78.796
37 36 NA 43 16 12 12 60.091
78 83 63 74 12.9 11 8 12.829
47 55 58 67 9.998 8 8 29.464
97 50 45 100 NA 12 14 NA
52 56 56 100 3 12 9 NA
44 23 39 100 NA NA 6 8.275
46 NA 49 NA NA 12 17 38.04
55 66 66 68 NA 12 11 4.429
47 52 63 66 NA NA 11 8.728
49 63 59 62 8 11 10 2.38
52 50 56 62 8 19 12 22.071
50 47 54 63 NA NA 11 1.162
52 66 NA 46 NA 15 14 29.126
56 69 50 75 NA 16 12 0.627
73 86 75 75 9.912 12 12 4.712
53 57 56 69 19 12 12 60.806
70 72 70 72 0 12 11 NA
57 55 55 53 NA 12 12 55.006
51 58 64 70 17 11 14 81.669
53 57 60 70 15.6 NA 11 30.261
50 70 70 72 18 13 12 32.471
56 55 65 70 NA NA 14 37.494
```

```
49 47 89 69 NA NA 10 21.307
65 72 65 76 NA 13 14 33.141
75 78 92 94 6.5 14 12 NA
68 71 56 75 NA 12 17 41.186
55 61 60 64 21 15 12 15.03
63 71 66 67 NA 13 12 77.568
56 50 0 59 1.257 10 9 9.566
62 67 63 57 30 18 16 69.921
73 48 NA 45 NA 10 12 68.913
62 53 73 65 30 12 11 NA
56 53 51 NA NA 11 NA 65.828
92 70 71 75 NA 11 12 43.517
NA 59 65 64 13 12 14 62.24
71 69 69 75 50 NA 14 68.115
86 82 90 91 NA 16 NA 51.648
86 87 92 94 20 17 18 87.545
NA 36 41 46 NA 17 16 35.874
58 54 65 64 NA 11 11 NA
69 71 87 81 NA NA 12 NA
84 35 29 26 NA 5 4 13.043
50 70 68 80 5 5 6 NA
NA 43 NA 48 NA 12 12 NA
76 59 73 89 20 NA 6 80.12
75 90 92 97 1.813 12 NA 62.376
43 43 71 46 10 13 13 79.836
40 44 40 40 10 13 13 27.214
48 48 80 84 0 NA 12 16.169
70 73 91 71 13 14 15 20.729
46 45 49 63 NA 12 12 28.229
38 58 66 65 NA NA 14 NA
49 46 66 65 21 3 3 NA
74 NA 70 85 30 NA 10 NA
46 75 78 74 34 NA 6 33.373
70 71 87 91 10 12 15 6.285
65 40 60 70 0 NA NA NA
61 54 56 55 NA NA NA NA
48 43 44 80 18.5 15 16 NA
75 81 NA 93 3 16 12 45.046
35 55 57 34 NA 6 6 2.354
73 72 84 84 NA 16 9 15.469
NA 39 62 33 16 8 14 43.394
63 93 81 89 NA NA 11 19.016
73 79 71 75 40 NA 10 10.487
63 84 75 64 40 NA 11 46.134
NA 66 55 66 NA 13 14 72.405
66 68 69 68 50 NA NA 1.239
61 57 38 43 0 NA 12 NA
63 53 90 85 NA NA NA 10.209
NA 54 NA 70 15 14 14 68.315
66 70 60 69 13.368 NA 12 5.745
73 94 92 95 24 NA NA 8.912
61 50 49 50 0 12 12 7.563
91 NA 94 NA 32 12 11 23.034
83 76 72 NA NA 9 9 3.123
```

```
51 57 NA NA NA NA NA 15.949
72 73 69 75 NA 11 8 NA
50 60 63 50 23.5 7 7 29.979
45 55 NA 53 NA NA 12 19.129
53 69 NA 1 NA 6 8 2.27
64 75 88 NA NA 14 12 NA
24 NA NA 45 NA 12 12 31.063
100 91 93 96 NA 13 13 NA
54 48 66 NA 23 16 14 1.646
51 49 49 99 23 16 14 10.343
62 89 91 90 NA 12 9 23.724
56 65 69 4 2.675 13 12 18.997
63 60 69 64 21 12 8 70.42
49 67 77 90 3 12 12 5.226
68 71 71 79 23 14 20 49.089
59 68 64 62 NA 17 18 50.701
55 59 68 71 41 11 14 NA
80 81 76 99 43 NA 10 NA
48 46 45 66 NA 18 16 42.198
60 68 69 70 10.4 12 12 NA
57 64 68 68 34 NA 11 5.428
63 73 70 57 10.5 12 11 18.953
51 60 53 51 2.328 NA 10 25.593
60 74 69 83 NA 12 11 24.889
53 58 61 61 8 12 12 57.984
60 62 57 60 NA 8 10 12.671
66 75 75 NA 13 12 14 29.434
53 38 36 NA NA 12 16 NA
47 37 99 98 1.7 12 12 NA
60 60 94 NA NA NA 8 3.085
69 NA 71 87 174.605 NA 3 NA
48 23 26 41 5.928 14 12 NA
54 38 53 NA 5.928 16 14 19.598
55 59 57 74 18 5 3 NA
65 80 68 72 NA NA 5 7.784
38 54 44 52 NA 8 6 15.716
38 28 54 66 NA NA 12 25.548
17 40 8 22 0 17 13 NA
25 26 70 18 0 17 13 NA
25 35 64 55 28 NA 7 3.832
NA 23 41 28 NA 12 13 NA
NA 73 62 85 NA 16 16 24.729
48 44 59 53 9.907 12 13 39.893
58 70 69 61 9 NA 14 10.666
80 88 87 93 NA 12 12 0.95
64 66 86 86 30 12 10 11.229
23 1 NA 55 NA NA NA NA
65 68 64 69 48 12 NA 6.724
85 73 58 64 18 14 12 2.009
55 67 69 77 18 16 17 69.876
45 52 56 52 NA 14 9 NA
31 19 19 50 NA 12 12 19.437
72 88 NA 82 18 11 14 NA
20 23 38 38 4 12 13 NA
```

```
56 68 72 70 2 5 11 7.295
28 33 52 NA NA NA 8 2.079
64 78 80 74 NA 12 11 34.026
60 54 66 67 12 9 12 10.8
44 38 47 65 2 NA 12 8.322
60 61 63 60 NA 11 12 26.82
54 62 69 51 NA 12 12 16.078
49 47 68 62 NA 12 12 26.185
44 49 47 47 NA NA 16 25.273
66 66 73 93 7.5 NA 11 NA
43 59 NA NA 27.648 12 12 NA
63 73 72 88 30 NA 11 NA
65 77 93 NA NA NA 12 NA
92 93 97 97 16 8 8 18.304
60 75 83 69 10 12 NA 8.453
59 68 73 75 NA 12 12 4.971
83 96 88 93 13 NA NA 22.839
20 61 58 21 NA NA NA 15.773
63 69 NA NA 14 12 9 8.82
76 81 85 67 NA 20 15 52.743
45 54 53 49 NA 11 12 NA
73 49 56 42 22 14 13 15.466
NA 46 59 57 27 NA 9 1.148
69 82 NA 72 NA 12 12 3.725
65 69 NA 79 8 12 12 5.925
42 44 47 50 20 12 11 0.316
61 68 72 73 NA NA 14 43.981
87 90 92 97 11 12 11 0.204
67 74 69 77 0.8 16 14 8.616
90 92 90 91 7 NA NA 46.574
65 59 51 NA 9.355 NA 12 26.175
NA 59 61 100 NA NA 13 69.753
85 90 89 90 NA NA 14 NA
70 69 64 78 83 12 12 46.931
59 75 69 70 18 12 12 NA
71 65 91 94 NA NA 12 NA
76 80 73 75 22 NA 12 24.204
51 54 51 55 NA 15 16 26.696
69 NA 80 71 0 14 15 22.92
48 57 60 59 16 NA 12 9.64
54 41 59 68 5 12 12 22.39
58 67 94 95 10 11 8 24.39
54 48 58 70 NA 14 11 1.518
64 54 52 35 NA NA 11 0
53 72 68 66 28.5 12 11 11.839
60 70 67 65 NA 10 11 10.045
31 NA 40 46 2 12 14 1.141
59 61 69 NA NA 11 11 10.267
59 50 100 61 NA 12 11 2.007
67 76 74 75 15 14 13 NA
62 92 NA NA NA NA 12 27.468
52 66 92 76 10 11 12 83.985
42 6 48 75 NA 7 8 NA
54 77 90 60 20 NA 11 NA
```

```
81 73 NA NA 46 14 12 68.089
34 36 44 54 NA NA 15 7.855
66 70 NA NA NA 12 10 23.616
86 56 70 NA NA 3 5 NA
66 72 79 NA NA 14 12 NA
49 73 70 78 0 NA 12 NA
62 50 61 81 NA 14 12 58.405
54 NA 64 64 NA 14 12 NA
93 90 97 98 27 11 10 10.541
22 20 NA 27 14 12 9 15.867
70 70 79 83 22 NA 14 46.848
81 NA NA 91 25 12 16 95.621
58 73 73 97 23 12 12 11.697
100 5 32 39 NA NA 12 38.443
95 98 97 97 16 NA 10 28.367
54 60 60 70 18.95 NA 12 NA
72 70 NA NA 5 13 10 21.795
100 49 61 56 1.7 12 12 NA
100 82 72 95 0 12 12 35.54
100 71 87 67 15 12 14 16.219
65 70 NA 50 NA 13 12 29.185
68 56 88 80 55 14 14 71.297
48 NA NA 71 NA 6 12 5.353
55 88 76 94 41 NA 12 1.535
78 77 88 88 15 4 9 22.874
86 91 91 88 15 NA NA 11.974
62 69 NA 49 NA NA 10 10.403
31 45 50 41 NA 12 11 21.898
46 64 NA 73 24 NA 13 NA
61 67 64 65 0.9 12 15 68.3
81 58 52 NA 17.056 NA NA 4.91
59 56 59 75 NA NA 15 NA
NA 66 57 71 NA 8 7 11.555
29 44 44 42 NA 12 12 29.784
49 52 54 62 14 12 12 22.651
43 78 70 90 17 10 12 44.454
83 68 76 71 27 NA 12 62.57
63 65 79 72 NA NA 12 15.447
55 59 59 58 NA NA 11 2.757
51 69 52 60 9 NA 10 NA
28 61 NA 59 NA 11 12 NA
44 41 75 83 15 12 11 65.985
58 56 67 66 15 12 11 NA
54 47 58 59 5 6 12 6.615
NA 40 55 68 NA NA NA 1.867
46 61 54 70 19 NA NA 3.256
44 56 66 54 3 3 NA NA
62 68 NA 81 10 16 12 NA
63 69 66 68 NA 15 12 9.809
61 59 61 64 7.2 NA 7 54.275
46 54 64 49 NA NA 12 3.302
NA 69 78 58 25 7 12 51.4
NA 57 60 74 25 NA 12 12.546
68 37 51 56 11 12 6 14.316
```

```
74  64  78  80  47     16  16  22.618
70  65  65  59  18     NA  6   54.423
83  73  91  93  65     12  12  3.654
74  70  53  64  1      9   12  13.816
59  56  80  57  20     12  NA  NA
72  NA  NA  49  NA     14  14  60.778
78  91  89  87  43     12  12  5.262
31  1   47  49  0.003  12  12  43.201
84  77  65  87  0.04   10  13  9.527
68  70  79  NA  8      NA  12  NA
77  NA  56  55  NA     3   11  NA
63  0   64  71  0.028  9   12  NA
42  50  57  57  NA     NA  10  2.011
31  37  57  41  NA     11  11  45.069
54  71  88  88  12     12  NA  16.714
54  NA  53  46  NA     12  14  44.382
78  NA  NA  35  NA     12  13  34.472
60  62  62  61  NA     4   9   NA
81  61  72  80  NA     12  12  75.4
88  90  70  92  25     9   9   45.651
61  63  63  60  NA     16  16  NA
43  43  54  48  NA     19  12  79.208
24  30  27  27  2.2    2   3   2.575
63  58  64  51  15.3   12  12  NA
79  80  82  87  26     12  14  7.54
46  48  63  68  7      NA  2   NA
57  55  50  65  4      NA  NA  6.33
66  69  76  69  NA     12  15  14.116
68  69  68  70  8.736  NA  NA  NA
59  60  64  61  15     NA  NA  NA
73  86  88  90  NA     NA  NA  NA
69  65  69  74  NA     NA  NA  NA
```

The following model is fitted to the data. Note that in the weight field, 0.1 (0 can also be used not to down-weight any data) is input to specify how much data to down-weight. In the control part, we add "mean.fit" to request the fit for mean and covariance.

The output is given below.

```
Descriptive statistics (N=422)
          Mean        sd  Min       Max  Skewness  Kurtosis  Missing Rate
piat1997  61.153  15.8735   16   100.000  -0.10876    3.2101      0.056872
piat1998  63.023  17.3987    0    98.000  -0.70221    4.1528      0.059242
piat1999  67.398  16.4878    0   100.000  -0.42875    3.6071      0.101896
piat2000  69.320  17.6567    1   100.000  -0.58754    3.6623      0.118483
income1   18.466  20.3267    0   174.605   4.09326   29.7891      0.414692
fgrade    12.152   2.9875    2    20.000  -0.75231    4.6546      0.312796
mgrade    12.189   5.0282    2    95.000  11.67801  195.0043      0.099526
asvab     30.496  25.1655    0    97.811   0.80760    2.5980      0.248815


Model information
Observed variables: piat1997 piat1998 piat1999 piat2000 .
Latent variables: F1 F2 .
The weight is: 0.1 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
```
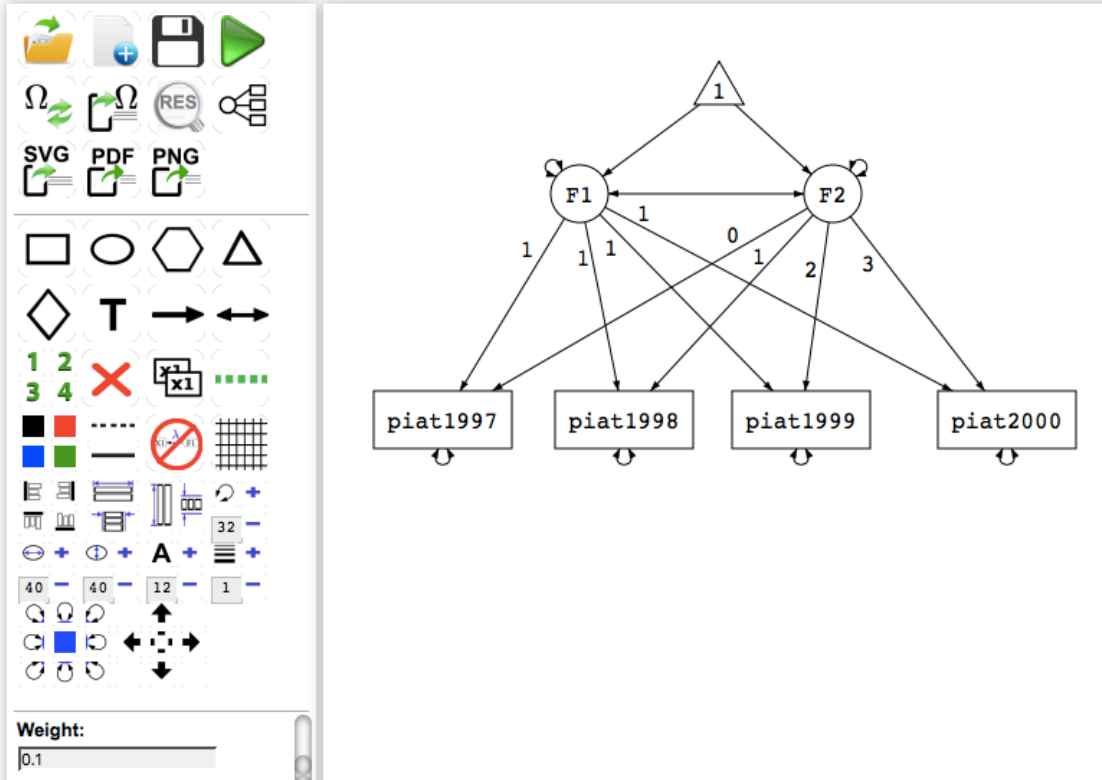
Figure 8: Path diagram for missing data and robust method

```
Fit for  both  mean  and  covariance
        Chi-square  statistic    13.836
        Degrees  of  freedom      5
        p-value          0.017
        Chi-square  statistic  (baseline  model)   5955.1
        Degrees  of  freedom  (baseline  model)     10
        CFI            0.999
        NFI            0.998
        NNFI           0.997
        RMSEA          0.058
Fit for  covariance  only
        Chi-square  statistic    11.907
        Degrees  of  freedom      3
        p-value          8e-03
        Chi-square  statistic  (baseline  model)   1024.7
        Degrees  of  freedom  (baseline  model)     7
        CFI            0.992
        NFI            0.988
        NNFI           0.98
        RMSEA          0.078
Fit for  mean  only
        Chi-square  statistic    1.934
        Degrees  of  freedom      2
        p-value          0.38
        Chi-square  statistic  (baseline  model)   216.56
        Degrees  of  freedom  (baseline  model)     3
```

```
            CFI                  1
            NFI                  0.991
            NNFI                 1
            RMSEA                0
Parameter Estimates
                       Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    piat1997             1.000
    piat1998             1.000
    piat1999             1.000
    piat2000             1.000
  F2 =~
    piat1997             0.000
    piat1998             1.000
    piat1999             2.000
    piat2000             3.000

Covariances:
  F1 ~~
    F2                  -5.060      3.954     -1.279     0.201

Intercepts:
    F2                   3.100      0.212     14.633     0.000
    F1                  61.010      0.716     85.264     0.000
    piat1997             0.000
    piat1998             0.000
    piat1999             0.000
    piat2000             0.000

Variances:
    F1                 170.729     15.570     10.965     0.000
    F2                   6.264      1.840      3.404     0.001
    piat1997            61.772      8.769      7.044     0.000
    piat1998            71.972      6.470     11.124     0.000
    piat1999            67.611      6.306     10.721     0.000
    piat2000            63.244      9.028      7.005     0.000



======================================
WebSEM ended at 16:50:00 on Oct 16, 2017
```

The output is given below.

```
Descriptive statistics (N=422)
          Mean      sd Min      Max Skewness  Kurtosis  Missing Rate
piat1997 61.153 15.8735  16  100.000 -0.10876    3.2101      0.056872
piat1998 63.023 17.3987   0   98.000 -0.70221    4.1528      0.059242
piat1999 67.398 16.4878   0  100.000 -0.42875    3.6071      0.101896
piat2000 69.320 17.6567   1  100.000 -0.58754    3.6623      0.118483
income1  18.466 20.3267   0  174.605  4.09326   29.7891      0.414692
fgrade   12.152  2.9875   2   20.000 -0.75231    4.6546      0.312796
mgrade   12.189  5.0282   2   95.000 11.67801  195.0043      0.099526
asvab    30.496 25.1655   0   97.811  0.80760    2.5980      0.248815
```
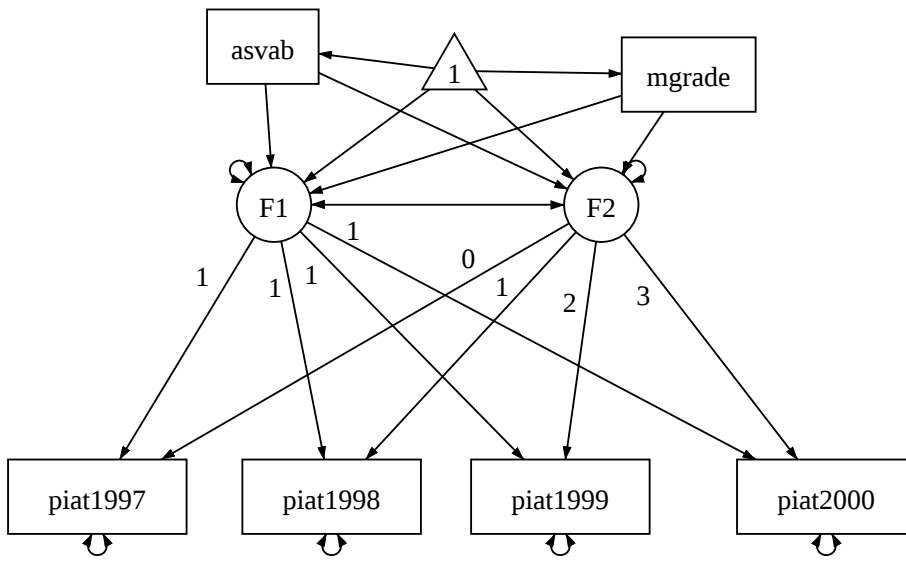
Figure 9: Path diagram for missing data and robust method

```
Model information
Observed variables: piat1997 piat1998 piat1999 piat2000 asvab mgrade .
Latent variables: F1 F2 .
The weight is: 0 .
The software to be used is: lavaan .
Test Statistics and Fit Indices
Fit for both mean and covariance
        Chi-square statistic    37.297
        Degrees of freedom      10
        p-value         0
        Chi-square statistic (baseline model)   6686.1
        Degrees of freedom (baseline model)     21
        CFI             0.996
        NFI             0.994
        NNFI            0.991
        RMSEA           0.077
Fit for covariance only
        Chi-square statistic    33.918
        Degrees of freedom      8
        p-value         0
        Chi-square statistic (baseline model)   834.66
        Degrees of freedom (baseline model)     18
        CFI             0.971
        NFI             0.959
        NNFI            0.929
        RMSEA           0.084
```

```
Fit for mean only
        Chi-square statistic   3.387
        Degrees of freedom     2
        p-value          0.184
        Chi-square statistic (baseline model)   145.05
        Degrees of freedom (baseline model)     3
        CFI              0.992
        NFI              0.977
        NNFI             0.985
        RMSEA            0.036
Parameter Estimates
                   Estimate   Std.err   Z-value   P(>|z|)
Latent variables:
  F1 =~
    piat1997           1.000
    piat1998           1.000
    piat1999           1.000
    piat2000           1.000
  F2 =~
    piat1997           0.000
    piat1998           1.000
    piat1999           2.000
    piat2000           3.000

Regressions:
  F1 ~
    asvab              0.092     0.030     3.075     0.002
  F2 ~
    asvab             -0.007     0.010    -0.718     0.472
  F1 ~
    mgrade             0.119     0.149     0.799     0.424
  F2 ~
    mgrade             0.017     0.050     0.344     0.731

Covariances:
  F1 ~~
    F2                -4.751     5.217    -0.911     0.362

Intercepts:
    F2                 2.998     0.726     4.127     0.000
    F1                56.417     2.158    26.140     0.000
    asvab             30.405     1.222    24.877     0.000
    mgrade            12.190     0.245    49.797     0.000
    piat1997           0.000
    piat1998           0.000
    piat1999           0.000
    piat2000           0.000

Variances:
    F1               170.582    17.551     9.719     0.000
    F2                 7.574     2.572     2.945     0.003
    piat1997          85.741    12.113     7.079     0.000
    piat1998         110.547     9.582    11.537     0.000
    piat1999          86.638     8.347    10.379     0.000
```

```
piat2000          105.491     13.187      8.000      0.000
asvab             628.869     43.345     14.509      0.000
mgrade             25.227      1.739     14.509      0.000


=====================================
WebSEM ended at 17:00:48 on Oct 16, 2017
```